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Competition in Markets with Demand Rigidity

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Kurzzusammenfassung

Diese Dissertation setzt sich aus fünf Forschungspapieren zusammen. Jedes Kapitel enthält ein Papier. Das erste Kapitel untersucht den Zusammenhang zwischen der Größe des Kundenstamms einer Firma und ihrem Gewinn in einem Markt mit Wechselkosten. Entgegen unserer Intuition wird gezeigt, dass Firmen nicht immer von einer Vergrößerung ihres Kundenstamms profitieren, weil diese die Intensität des Wettbewerbs beeinflusst. Kapitel 2 führt eine ähnliche Untersuchung durch, aber für einen Markt, in dem die Konsumenten unvollständig über die Standorte der Anbieter informiert sind. Es zeigt sich auch hier, dass eine Firma nicht immer von einem großen Kundenstamm profitiert. Die zugrunde liegenden Mechanismen unterscheiden sich jedoch deutlich von denen in Kapitel 1. Kapitel 3 ist eine Erweiterung des Modells mit unvollständiger Konsumenteninformation hin zu einer vollständig dynamischen Version. Im Zentrum der Analyse stehen nun die dynamischen Eigenschaften des Modells. Unter den Annahmen über die graduelle Verbreitung von Information auf der Konsumentenseite entsteht Trägheit in den Marktanteilen der Firmen. Dynamik entsteht im Modell ausschließlich aufgrund der Verwendung von gemischten Preisstrategien. Kapitel 4 analysiert Wettbewerb in einem vertikal differenzierten Markt. Hier gibt es keine Trägheit auf der Nachfrageseite. Das Hauptergebnis der Analyse ist, dass Wohlfahrtsverluste, die im Duopol aus ineffizienter Qualitätswahl resultieren, in Märkten mit drei oder mehr Wettbewerbern fast vollständig verschwinden. Dieses überraschende Ergebnis resultiert aus einem Regimewechsel in der Art des Wettbewerbs, der beim Übergang vom Duopol zum Markt mit drei Wettbewerbern auftritt. Kapitel 5 ist eine Erweiterung von Kapitel 4. Während in Kapitel 4 ein quadratischer Zusammenhang zwischen Kosten bzw. Zahlungsbereitschaft und Qualität angenommen wurde, wird die Analyse nun für eine allgemeinere nicht-lineare Abhängigkeit durchgeführt. Es werden grundlegende Einsichten über das Funktionieren von vertikal differenzierten Märkten vermittelt. So zeigt sich, dass der allgemein postulierte Vorteil der Firma mit der höheren Produktqualität nicht allgemeingültig ist. Ob dieser besteht, hängt von der Art der strategischen Interaktion ab.

Schlagwörter: Kundenstamm, Wechselkosten, unvollständige Information, Marktanteile, vertikale Differenzierung

Abstract

This dissertation consists of five independent research papers. Each chapter represents one paper. The first chapter analyzes the shape of the relation between the size of a firm's customer base and profit in a market with consumer switching costs. Contrary to common wisdom, it is shown that a firm is not automatically better off with a larger customer base, as the size of its customer base affects the intensity of price competition. Chapter 2 performs a similar exercise, but for a market where consumers are not fully informed about the locations of the different suppliers. Once more, it is shown that firms do not always benefit from an increase in the size of their customer base. However, the underlying mechanisms are rather different than in the model with switching costs. Chapter 3 is an extension of the model introduced in chapter 2 to a fully dynamic game. The focus of chapter 3 is on the dynamics in a market with incomplete consumer information. Under the assumptions about the gradual diffusion of information among consumers, there is inertia in the market shares. Dynamics are generated solely by the firms' usage of mixed pricing strategies. Chapter 4 analyzes competition in a vertically differentiated market. There is no inertia on the demand side. The main result of the analysis is, that welfare losses that stem from an inefficient choice of qualities in the duopoly case, disappear almost completely as soon as three or more competitors are in the market. This surprising result is related to a regime change in the nature of competition that occurs at the transition from duopoly to triopoly. Chapter 5 is an extension of chapter 4. Whereas the model introduced in chapter 4 was based on a quadratic relation between costs or willingness-to-pay and quality, the analysis is now extended to a more general non-linear dependency. The analysis provides fundamental insights into the functioning of vertically differentiated markets. Interestingly, the well-known high-quality advantage is not a robust feature of these markets. Whether it is obtained, depends on the nature of strategic interaction between the firms.

Keywords: customer base, switching costs, incomplete information, market share, vertical differentiation

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Allgemeine Einführung

In der ökonomischen Literatur wurde lange Zeit angenommen, dass Konsumenten über alle in einem Markt verfügbaren Produkte vollständig informiert sind, und dass sich die Nachfragemengen zu jedem Zeitpunkt vollständig an die gegebenen Preise anpassen. Wenn sich ein Preis oder eine Qualität ändert, oder ein neues Produkt eingeführt wird, dann müssten sich demnach die Marktanteile der Firmen sofort an die neuen Bedingungen anpassen. Es dürfte für das Marktergebnis also keine Rolle spielen, ob eine Firma gegenwärtig über einen großen oder einen kleinen Kundenstamm[1] verfügt. Beggs und Klemperer (1992) schreiben jedoch: „Managers often seem more concerned with market shares than with short-run profits." Es hat demnach den Anschein, als würde der Aufbau eines großen Kundenstamms für die langfristige Positionierung von Firmen wichtig sein. Belege dafür gibt es viele.[2] So lautet z.B. eine aktuelle Schlagzeile aus der Internetpresse: „Apple faces iPhone profit or market share dilemma".[3] Im Beitrag wird u.a. auf Sacconaghi (ein bekannter Analyst) Bezug genommen: „The analyst, however, said that Apple would be unlikely to sustain the same level of profitability if it expects to sell 10 million phones in 2008, as its CEO Steve Jobs had hoped. Sacconaghi noted that carrier payments to the company could allow it 'to pull some strategic strings' to drive up unit sales, but only at the expense of profitability."

Ein großer Kundenstamm kann für die zukünftige Profitabilität einer Firma nur dann bedeutsam sein, wenn die Konsumenten nach einer Preisänderung nicht sofort bzw. nicht alle zum billigeren Anbieter wechseln.[4] Dieses Verhalten der Nachfrage wird im Folgenden als *Nachfragerigidität* bezeichnet. Eine mögliche Begründung für Nachfragerigidität ist, dass den Konsumenten beim Wechsel des Anbieters Kosten entstehen. Demnach wird ein Konsument nur dann den Anbieter wechseln, wenn der Preisunterschied die Wechselkosten aufwiegt.[5] Es kann zwischen zwei Arten von Wechselkosten unterschieden werden. Erstens: „natürliche" Wechselkosten, die z.B. durch das Erlernen der Bedienung eines neuen Produktes entstehen (Beispiel: Wechsel von Microsoft- zu Apple-Software), oder aufgrund von Transaktionskosten (Beispiel: Eröffnung eines neuen Bankkontos). Und zweitens:

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[1] hier: „Kundenstamm" = Verkaufsmenge in der Vergangenheit

[2] Siehe z.B. Allenby und Lenk (1995), Antunano und Kuo (2002), Dorward (1977), Kirman und Vriend (2001), Odin, Odin und Valette-Florence (2001), Shum, (2004), Stern und Hammond (2004), Yim und Kannan (1999).

[3] http://www.macnn.com/articles/07/10/10/aapl.target.up.by.30/

[4] Hier wird der Einfachheit halber von homogenen Gütern ausgegangen. Netzwerkeffekte werden nicht betrachtet.

[5] Für einen Überblick über die Wechselkostenliteratur, siehe z.B. Klemperer (1995).

„künstliche" Wechselkosten, die z.B. durch Rabatte bei Wiederholungskäufen entstehen (Beispiel: Vielfliegerprogramme).

Ein Aspekt, der in der Literatur bisher wenig beachtet wurde, ist, dass auch unvollständige Konsumenteninformation zu rigider Nachfrage führen kann. Diesem Aspekt soll in dieser Arbeit besondere Aufmerksamkeit geschenkt werden. Der Effekt entsteht, wenn Konsumenten im Markt als Wiederholungskäufer auftreten, und über Produkte, die sie in der Vergangenheit gekauft haben, besser informiert sind als über die übrigen Produkte.[6] Demnach haben Firmen mit einem großen Kundenstamm einen potenziellen Vorteil gegenüber ihren Konkurrenten, da viele Konsumenten über ihr Angebot informiert sind.

Die Analyse von Märkten mit rigider Nachfrage bildet das Hauptthema dieser Arbeit. Eine der Fragen, die beantwortet werden sollen, ist, wann Firmen von einer Vergrößerung ihres Kundenstamms profitieren können. Die Antwort auf diese Frage ist nicht trivial, denn die Marktaufteilung beeinflusst die Intensität des Preiswettbewerbs. Demnach kann eine Vergrößerung des Kundenstamms einer Firma deren Gewinn schmälern, wenn dadurch der Preiswettbewerb intensiver wird.

Die Arbeit beschäftigt sich darüber hinaus mit Marktanteilsdynamiken bei unvollständiger Konsumenteninformation. Aufgrund der daraus resultierenden Nachfragerigidität sind die Marktaufteilungen durch ein gewisses Maß an zeitlicher Stabilität charakterisiert. Für ein solches Verhalten gibt es auch Belege aus der Praxis.[7] Nachfragerigiditäten sind eine mögliche Erklärung dafür, warum einzelne Firmen Märkte z.T. über längere Zeit dominieren, während kleinere Konkurrenten ihren Marktanteil nur langsam erhöhen können.[8]

Ein weiteres Themengebiet dieser Arbeit sind Märkte mit vertikal differenzierten Gütern (das sind Güter mit unterschiedlicher Qualität). Doch bevor der Inhalt der Arbeit im Detail vorgestellt wird, folgt ein kurzer Rückblick zur Entwicklung der Idee von Nachfragerigiditäten.

So erwähnt z.B. Bain in seiner empirischen Analyse von 1956, dass Markteindringlinge ohne Kundenstamm im Wettbewerb mit etablierten Anbietern benachteiligt sein können. Dafür nennt er verschiedene Gründe: „(a) customer inertia, habit, and loyalty…; (b) preferences for

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[6] Dazu zählt z.B. Kenntnis über die Existenz eines Produktes bzw. des entsprechenden Anbieters, sowie Wissen über den Standort des Anbieters und die Eigenschaften des Produktes.

[7] Siehe z.B. Pakes (1987).

[8] Ein anschauliches Beispiel dafür ist im deutschen Getränkemarkt zu finden. Die „Bionade" – ein Getränk aus biologischer Produktion, konnte ihren Marktanteil über die letzten Jahre graduell ausweiten. Das Produkt war den Konsumenten zunächst unbekannt, und wurde lange Zeit vorwiegend durch Mundpropaganda verbreitet.

products of established firms based on advertising or on established product “reputations“; (c) allegiances to established firms based on services supplied to customers; and (d) established dealer systems, owned or more or less controlled by established firms.“ Bain schreibt weiter: „The inability of potential entrants to match the physical product designs of established firms does not stand out as generally important in industries within this sample, but the allegiance of consumers to established products in areas in which they are ignorant or uncertain concerning the actual properties of products is quite important.“

Kaldor (1935) führt den Begriff der „buyers’ inertia“ ein, zu der er u.a. folgendes schreibt: „No doubt, in most cases, the products of various producers selling the same sort of goods are not ‚perfect substitutes’… The reasons for such ‚market imperfection’ may be classed under one of three headings. There may be slight differences in the products themselves…; or differences in the geographical location of producers…; or finally, there may exist a certain ‚inertia’ on behalf of the buyers themselves who will require either some time, or a certain magnitude in the price-difference, before they make up their minds to buy from another seller – even if they are quite indifferent as between the products of different sellers.“

In eine ähnliche Richtung zielt auch Selten’s berühmte Arbeit von 1965, in der er einen Oligopolmarkt mit „Nachfrageträgheit“ anhand eines theoretischen Modells analysiert. Unter Nachfrageträgheit versteht der Autor, dass Nachfrager nicht sofort auf Preisänderungen reagieren. Dafür führt er u.a. folgende Gründe an: Preisänderungen werden nicht sofort bemerkt, Einkaufsgewohnheiten werden nur langsam geändert, mangelnde Markttransparenz, Starrheit der Absatzwege, Kundentreue. Bei Selten spielt die zeitliche Dimension bei der Änderung von momentan vorhandenen Marktanteilen eine besondere Rolle, während (anders als bei Bain) Marktzutritt nicht berücksichtigt wird. Selten schreibt: „Auch die Produktdifferenzierung kann zur Nachfrageträgheit beitragen, indem sie die Umstellung von dem Produkt eines Anbieters auf das Produkt eines anderen Anbieters erschwert.“ Dabei denkt der Autor vornehmlich an Märkte, die er als „quasihomogen“ bezeichnet: „Die Quasihomogenität besagt, daß in einer Situation, in der die Gesamtnachfrage konstant ist, die Marktaufteilung nur dann unverändert bestehen bleiben kann, wenn alle Anbieter den gleichen Preis haben, weil Preisunterschiede immer eine Verschiebung der Marktanteile zugunsten der billigeren Anbieter zur Folge haben. … Die Produkte der einzelnen Anbieter müssen trotz ihrer Differenzierung noch ungefähr gleichwertig sein und die Unterschiede zwischen den Produkten dürfen lediglich eine Verstärkung der Nachfrageträgheit zur Folge haben.“ Es ist bemerkenswert, dass Selten mit seinem Konzept der Nachfrageträgheit u.a. der Wechselkostenliteratur knapp zwei Jahrzehnte voraus war. Auch wenn er Wechselkosten nicht als solche definiert hat, beinhaltet sein Konzept letztlich die Idee davon: Konsumenten wechseln langsam bzw. nur teilweise zu einem günstigeren Anbieter, weil die Produkte aus ihrer Sicht differenziert sind. Allerdings sind die Produkte nicht im Hinblick auf ihre Funktionalität verschieden; die Differenzierung erschwert lediglich den Wechsel von einem Anbieter zu einem anderen. Selten motiviert sein Konzept der Nachfrageträgheit aber auch mit unvollständiger Konsumenteninformation, ein Aspekt, der in der hier vorliegenden Arbeit von zentraler Bedeutung ist.

Es sollte jedoch nicht unerwähnt bleiben, dass Selten's Arbeit weniger aufgrund des eigentlichen Themas der Arbeit, also der Analyse von Märkten mit Nachfrageträgheit, sondern viel mehr durch die Einführung des Konzepts vom teilspielperfekten Gleichgewicht berühmt wurde. Dass die Nachfrageträgheit als solche weitgehend in Vergessenheit geraten ist, und nur gelegentlich von anderen Autoren erwähnt wurde, hat verschiedene Ursachen. Einerseits war der Autor mit seinem Beitrag möglicher Weise zu weit seiner Zeit voraus, um damit einen neuen Forschungsstrang zu begründen, und seine Analyse war so umfassend, dass sie wenig Platz für Modifikationen oder Erweiterungen ließ. Andererseits ist die Formalisierung seines Konzepts der Nachfrageträgheit weitestgehend *ad hoc*, d.h., es fehlt eine schlüssige mikroökonomische Begründung für seine Modellannahmen. So lässt der Autor z.B. offen, ob sich die Nachfrage einer Firma ableitet von unterschiedlichen Vorbehaltspreisen der Konsumenten für ein unteilbares Gut, oder ob möglicher Weise alle Konsumenten die gleichen Präferenzen haben und die nachgefragte Menge pro Konsument sinkt im Preis. Auch die Möglichkeit der Koexistenz von trägen und nicht-trägen Konsumenten im Markt bleibt unerwähnt. Bei funktional identischen Gütern würde die Anwesenheit von nicht-trägen Konsumenten (also vollständig informierter Konsumenten ohne Wechselkosten) die Existenz eines dynamischen Preisgleichgewichts in reinen Strategien, wie Selten es berechnet hat, ausschließen.

Ähnlich wie bei Selten's Modell ist auch der Ausgangspunkt dieser Arbeit die Annahme, dass Nachfragemengen von vergangenen Marktergebnissen abhängen. Die Größe des Kundenstamms einer Firma beeinflusst demnach deren Profitabilität. Die Arbeit gliedert sich in drei Haupt-Themenbereiche. Der erste Themenbereich sind Märkte mit vollständig informierten Konsumenten und Wechselkosten. In solchen Märkten wechseln Konsumenten mit unendlicher Geschwindigkeit den Anbieter (falls sie wechseln), aber ein gegebener Preisunterschied führt i.A. nicht dazu, dass alle Konsumenten wechseln. Der zweite Themenbereich sind Märkte mit unvollständig informierten Konsumenten (ohne Wechselkosten). Die Rigidität der Nachfrage hat hier eine zeitliche Dimension: bei gegebenen

Preisunterschieden verschieben sich die Marktanteile der Firmen mit endlicher Geschwindigkeit. Der dritte Themenbereich konzentriert sich auf vertikal differenzierte Märkte. Rigiditäten der Nachfrage spielen hier nur indirekt eine Rolle. So kann gezeigt werden, dass das gängige Standard-Modell mit vollständig informierten Konsumenten nicht in der Lage ist, bestimmte empirische Beobachtungen zu erklären (s.u.). Demnach müssen in der Praxis Marktunvollkommenheiten eine Rolle spielen, die vom Modell nicht erfasst werden. Die plausibelste Erklärung ist unvollständige Konsumenteninformation, die zu Rigiditäten führen kann, wie jene, die im Rahmen des zweiten Themenbereiches betrachtet werden. Diese werden für vertikal differenzierte Märkte jedoch nicht explizit untersucht.

Diese drei Haupt-Themenbereiche werden in fünf Kapiteln behandelt, von denen sich jedes auf eine bestimmte Fragestellung konzentriert. Jedes Kapitel kann als eine separate Arbeit betrachtet werden. Das führt an einigen Stellen zu Redundanzen bei der Einführung der Modellannahmen, hat jedoch den Vorteil, dass die Kapitel unabhängig von einander gelesen werden können. Die Inhalte der fünf Kapitel werden im Folgenden zusammengefasst.

Kapitel 1 und 2 beschäftigen sich mit der Frage, wann Firmen von einer Vergrößerung ihres Kundenstamms profitieren können. Kapitel 1 behandelt diese Frage im Rahmen eines Wechselkostenmodells. Für Märkte mit Wechselkosten wird i.A. angenommen, dass Firmen von einem großen Kundenstamm profitieren. Klemperer (1995) weist jedoch darauf hin, dass diese Aussage nicht allgemeingültig ist, denn eine Vergrößerung des Kundenstammes einer Firma kann ein aggressiveres Preissetzungsverhalten der Konkurrenten zur Folge haben, was zumindest theoretisch dazu führen kann, dass die Firma dann schlechter gestellt ist als zuvor. Anhand eines Duopolmodells wird im Kapitel 1 gezeigt, dass ein solches Resultat keine rein theoretische Möglichkeit darstellt, sondern ein empirisch relevantes Ergebnis sein kann. Diese Schlussfolgerung ergibt sich daraus, dass das Ergebnis unter plausiblen Modellannahmen erzeugt wird. Dazu zählen insbesondere homogene Güter und gleichmäßig verteilte Wechselkosten. In der funktionalen Abhängigkeit zwischen der Größe des Kundenstamms und dem Gewinn einer Firma ergibt sich ein Maximum bei einer *ex-ante* gleichförmigen Marktaufteilung. Das hängt damit zusammen, dass der Preiswettbewerb am intensivsten ist, wenn der Markt *ex-ante* asymmetrisch aufgeteilt ist. Darüber hinaus wird gezeigt, dass die Vorhersage, dass Wechselkosten i.A. zu einer Abschwächung des Wettbewerbs führen, im verwendeten Modellrahmen verlässlicher ist als in bisher bekannten Wechselkostenmodellen.

Kapitel 2 beschäftigt sich ebenfalls mit der Frage, wann Firmen von einer Vergrößerung ihres Kundenstamms profitieren können, jedoch im Rahmen eines Modells mit unvollständig informierten Konsumenten, die als Wiederholungskäufer auftreten. Auch hier ist zu beachten, dass ein großer Kundenstamm nicht automatisch vorteilhaft für eine Firma sein muss, wenn dieser zu aggressiverem Preissetzungsverhalten der Konkurrenten führt. Anhand eines Duopolmodells, in dem Konsumenten aktiv suchen oder Mundpropaganda betreiben können, wird gezeigt, dass der Zusammenhang zwischen der Größe des Kundenstamms und dem (erwarteten) Gewinn einer Firma nicht immer monoton steigend ist, sondern vielfältiger Gestalt sein kann. Bei nahezu vollständiger Information ergibt sich eine V-förmige Relation. Demnach ist kein Kundenstamm, bzw. ein sehr großer Kundenstamm für eine Firma günstiger als ein mittelgroßer Kundenstamm. Das liegt daran, dass der Preiswettbewerb am intensivsten ist, wenn der Markt *ex-ante* symmetrisch aufgeteilt ist. Rigidität, die aus unvollständiger Konsumenteninformation entsteht, führt demnach zu einem Ergebnis, das dem unter der Annahme von Wechselkosten erhaltenen Ergebnis entgegengesetzt ist: dort ist der Preiswettbewerb am wenigsten intensiv bei einer *ex-ante* symmetrischen Aufteilung des Marktes! In Kapitel 2 wird darüber hinaus eine Endogenisierung der Mundpropaganda-Entscheidung eingeführt, und das Modell wird für größere Firmenzahlen verallgemeinert.

Kapitel 3 basiert auf dem selben Grundmodell wie Kapitel 2, erweitert dieses jedoch zu einem unendlich-periodigen Preisspiel mit rigider Nachfrage. Allerdings werden nun die *dynamischen* Eigenschaften des Modells näher untersucht. Eine zentrale Eigenschaft des Modells besteht nämlich darin, dass die Marktaufteilung zwischen den Firmen keinem stabilem Zustand zustrebt, sondern dass die Marktanteile unentwegt fluktuieren. Die zentrale Triebfeder hinter diesen Fluktuationen ist das Nicht-Vorhandensein eines Gleichgewichts in reinen Strategien. Stattdessen würfeln die Firmen ihre Preise gemäß einer im Modell zu bestimmenden Verteilungsfunktion, die von der gegenwärtigen Marktaufteilung abhängig ist. Die resultierende *Preisdispersion* ist eine wohlbekannte Eigenschaft von Standard-Modellen aus der Suchliteratur.[9] Allerdings haben Autoren von entsprechenden Arbeiten aus diesem Bereich der Marktanteilsdynamik, die sich bei Preisdispersion zwangsläufig ergibt, bisher wenig Beachtung geschenkt. Nach meinem Wissen ist die Analyse in Kapitel 3 in dieser Hinsicht neuartig. Es sollte ebenfalls erwähnt werden, dass die Möglichkeit von Wiederholungskäufen in der bisherigen Suchliteratur so gut wie keine Rolle gespielt hat. Ohne Wiederholungskäufe gibt es trotz unvollständiger Konsumenteninformation natürlich

[9] Das Vorhandensein von Preisdispersion wird in einigen empirischen Arbeiten belegt. Lach (2002) schreibt z.B.: „I find that price dispersion prevails even after controlling for observed and unobserved product heterogeneity. Moreover, intra-distribution mobility is significant: stores move up and down the cross-sectional price distribution. Thus, consumers cannot learn about stores that consistently post low prices… price dispersion persists over time as predicted by Varian's (1980) model of sales."

keine Rigidität in der Nachfrage. Deshalb entspricht die „Dynamik“ eines Standard-Suchmodells der simplen Wiederholung eines Einperiodenspieles. Im Gegensatz dazu gibt es im hier betrachteten Modell einen direkten Zusammenhang zwischen der Vergangenheit des Marktes und dem gegenwärtigen Marktergebnis. Es sollte betont werden, dass – anders als bei vielen Modellen aus dem makroökonomischen Bereich – die Dynamik des Modells nicht von exogenen Schocks abhängig ist, sondern allein aus dem Streben der Firmen nach Gewinnmaximierung resultiert. Ziel der Analyse ist es, diese Dynamik zu charakterisieren, und die Triebkräfte dahinter aufzudecken.

Kapitel 4 und 5 beschäftigen sich mit vertikal differenzierten Märkten. Hier spielen Rigiditäten der Nachfrage nur indirekt eine Rolle. Es ist allgemein bekannt, dass es in vertikal differenzierten Märkten mit vollständig informierten Konsumenten zu übermäßiger Qualitätsspreizung kommt, weil die Firmen dadurch die Intensität des Preiswettbewerbs abschwächen können. Diese aus der theoretischen Literatur stammende Vorhersage hat Autoren dazu veranlasst, sich mit wohlfahrtssteigernden Maßnahmen zu befassen, insbesondere Mindest-Qualitätsstandards. In Kapitel 4 wird die Allgemeingültigkeit dieser Ergebnisse in Frage gestellt, indem überprüft wird, ob die Maßnahmen auch in Märkten mit mehr als zwei Wettbewerbern wohlfahrtssteigernd wirken können. So wird insbesondere gezeigt, dass die Überdifferenzierung fast vollständig verschwindet, sobald mindestens drei Firmen im Markt aktiv sind. Demnach muss nahezu jede Form von Regulierung wohlfahrtsmindernd wirken, denn freier Wettbewerb führt zu einem annähernd optimalen Ergebnis. Es sollte also eine *laissez-faire* Politik betrieben werden. Diese Aussage steht im Widerspruch zu dem, was in der Realität beobachtet wird. Mindest-Qualitätsstandards sind in der Praxis nämlich weit verbreitet, und ihre Verwendung beschränkt sich keineswegs auf Duopolmärkte. Unter der Annahme, dass Mindest-Qualitätsstandards nicht generell als Zeichen von Überregulierung gedeutet werden können, folgt daraus, dass es weitere Ursachen für Marktversagen geben muss, die von dem Standard-Modell nicht erfasst werden. Die plausibelste Erklärung ist, dass Konsumenten die Qualität der Güter nicht genau einschätzen können. Demnach liefert die Analyse in Kapitel 4 ein starkes Argument für das Vorhandensein von unvollständiger Konsumenteninformation.

In Kapitel 5 wird lediglich der Duopolfall eines vertikal differenzierten Marktes mit vollständig informierten Konsumenten betrachtet. Es wird gezeigt, dass eine allgemein anerkannte Vorhersage für solche Märkte, nämlich die, dass der Anbieter des Produktes mit der höheren Qualität den höheren Gewinn erzielt, falsch ist. Statt dessen kann der Anbieter mit der geringeren Qualität den höheren Gewinn im Markt erzielen, wenn die Beziehung zwischen Nutzen und Qualität konkav, oder die Beziehung zwischen Stückkosten und Qualität konvex ist (oder beides). Beides sind plausible Annahmen. Welche Firma letztlich den höheren Gewinn erzielt, hängt davon ab, wie die Firmen unter den jeweiligen Modellannahmen strategisch interagieren.

Die Ergebnisse dieser Arbeit legen nahe, dass ein „profit or market share dilemma“, wie jenes, in dem Apple sich z.Zt. möglicher Weise befindet, auch aus theoretischer Sicht plausibel ist. Wenn die Nachfrage rigide ist, kann es für eine Firma sinnvoll sein, den Preis zu senken oder in Werbung zu investieren und damit auf kurzfristige Gewinne zu verzichten, um einen größeren Kundenstamm aufzubauen. Abhängig davon, wie die Konkurrenten auf solche Maßnahmen reagieren, kann der größere Kundenstamm für die zukünftige Profitabilität der Firma wichtig sein.

General Introduction (English)

In the economics literature, it has long been assumed that consumers are fully informed about all products available in a market, and that firms' demands fully adjust to the prices at each instant. Hence, if a price or a quality level changes, or if a new product is introduced, then market shares should immediately adapt to the new conditions. It should not play a role for the market outcome whether a firm currently has a large or a small customer base[10]. Beggs and Klemperer (1992), however, write: "Managers often seem more concerned with market shares than with short-run profits.“ It, thus, seems as if a large customer base can be important for the future profitability of a firm. There is a lot of evidence for this.[11] A headline from the current internet press reads: „Apple faces iPhone profit or market share dilemma“.[12] The article refers to Sacconaghi (a well-known analyst): „The analyst, however, said that Apple would be unlikely to sustain the same level of profitability if it expects to sell 10 million phones in 2008, as its CEO Steve Jobs had hoped. Sacconaghi noted that carrier payments to the company could allow it 'to pull some strategic strings' to drive up unit sales, but only at the expense of profitability.“

[10] here: "customer base" = past sales volume

[11] See e.g. Allenby and Lenk (1995), Antunano and Kuo (2002), Dorward (1977), Kirman and Vriend (2001), Odin, Odin and Valette-Florence (2001), Shum, (2004), Stern and Hammond (2004), Yim and Kannan (1999).

[12] http://www.macnn.com/articles/07/10/10/aapl.target.up.by.30/

A large customer base can only be important for the future profitability of a firm if consumers do not immediately or not completely switch to the cheaper supplier after a price change.[13] This demand behavior is in the following referred to as *demand rigidity*. A possible explanation for demand rigidity is that consumers incur costs when switching the supplier. Therefore, a consumer will only switch if the price differential outweighs the switching costs.[14] There are two types of switching costs. Firstly, "natural" switching costs, that may e.g. be due to the effort of learning how to use a new product (for example when switching from Microsoft to Apple software), or due to transaction costs (for example when opening a new bank account). And secondly, "artificial" switching costs, that may e.g. be due to discounts for repeat purchasers (example: frequent-flyer programs).

An aspect that has received rather little attention in the literature is that incomplete consumer information can also induce demand rigidity. This aspect will be a major focus of this work. The effect arises when consumers are repeat purchasers who possess more information about products they have previously purchased[15] than about the other products in a market. Therefore, firms with a large customer base have a potential advantage over their competitors, as many consumers are informed about their offer.

The analysis of markets with rigid demand is the main topic of this work. Among the questions that shall be analyzed is, under what conditions firms can benefit from an increase in the size of their customer base. The answer to this question is not trivial because the market split affects the intensity of price competition. Therefore, an increase in the size of a firm's customer base can reduce the firm's profit if it induces fiercer price competition.

Furthermore, this work analyzes market share dynamics under incomplete consumer information. Due to the resulting demand rigidity, market splits are characterized by a certain degree of temporal stability. There is empirical evidence for such behavior.[16] Demand rigidities are a possible explanation for why certain firms can dominate markets for extended periods of time, while their smaller competitors can only gradually achieve a higher market share.[17]

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[13] Here, I assume homogeneous goods for simplicity. Network effects are not considered.

[14] For an overview over the switching cost literature, see e.g. Klemperer (1995).

[15] This information may be knowledge about the existence of a product or the respective supplier, about the supplier's location or the product's characteristics.

[16] See e.g. Pakes (1987).

[17] A nice example for this can be found in the German market for soft drinks. "Bionade" – an organic soft drink – could gradually increase its market share over the past few years. The product was originally unknown to the consumers and gradually became more famous via word-of-mouth.

Another topic of this work are markets with vertically differentiated goods (these are goods that differ in quality). But before the contents of this work is described in more detail, let us briefly look back on the development of the idea of demand rigidity.

In his empirical analysis from 1956, Bain points out that entrants with no customer base may be at a disadvantage relative to established firms. He mentions several reasons for this: "(a) customer inertia, habit, and loyalty…; (b) preferences for products of established firms based on advertising or on established product "reputations"; (c) allegiances to established firms based on services supplied to customers; and (d) established dealer systems, owned or more or less controlled by established firms." He further writes: "The inability of potential entrants to match the physical product designs of established firms does not stand out as generally important in industries within this sample, but the allegiance of consumers to established products in areas in which they are ignorant or uncertain concerning the actual properties of products is quite important."

Kaldor (1935) introduces the term "buyers' inertia". He writes: "No doubt, in most cases, the products of various producers selling the same sort of goods are not 'perfect substitutes'… The reasons for such 'market imperfection' may be classed under one of three headings. There may be slight differences in the products themselves…; or differences in the geographical location of producers…; or finally, there may exist a certain 'inertia' on behalf of the buyers themselves who will require either some time, or a certain magnitude in the price-difference, before they make up their minds to buy from another seller – even if they are quite indifferent as between the products of different sellers."

Selten's seminal work from 1965 goes in a similar direction. In a theoretical model, he analyzes an oligopolistic market characterized by "demand inertia". This means that consumers do not immediately react to price changes. The author mentions the following reasons for this: price changes are not immediately discovered by the consumers, shopping habits change only gradually, a lack of market transparency, rigidity of sales channels, customer loyalty. In Selten's work, the intertemporal dimension in the adaptation of market shares to price changes plays a crucial role, while (in contrast to Bain) the possibility of entry is neglected. According to Selten, product differentiation can also contribute to demand inertia by making it more difficult to switch from one supplier's product to the product of another supplier. Selten focuses on markets that he refers to as "quasi-homogeneous". According to him, quasi-homogeneity means that, in situations where aggregate demand is constant, the market split only changes when firms choose non-identical prices for their products. Therefore, the products of the different suppliers should – despite their differentiation – be equally valuable, and the differences between them should only lead to more demand inertia. It is remarkable that, with his concept of demand inertia, Selten was almost two decades ahead of the switching cost literature. Although he did not explicitly define switching costs, he essentially had the idea of them in mind: consumers switch only gradually or incompletely to a cheaper supplier, because the products are, from their perspective, differentiated. However, products are not functionally different; the differentiation only makes it harder to switch from one supplier to another. Selten further motivates his concept of demand inertia by incomplete consumer information – an aspect that plays a major role also in this work.

It should, however, be mentioned that Selten's work became mostly famous for the introduction of the concept of the subgame perfect equilibrium, and not for its central theme, namely the analysis of markets with demand inertia. The latter received rather little attention in the literature, which may be due to the following reasons. On the one hand, with his concept of demand inertia, the author was perhaps too far ahead of his time to initiate a new strand of literature, and his analysis was too comprehensive to leave space for modifications or extensions. On the other hand, his formalization is mostly *ad hoc*, and lacks a convincing microeconomic foundation. E.g., the author does not clarify whether a firm's demand results from different reservation prices of the consumers for an unseparable good, or whether consumers have possibly identical preferences, and each consumer's demand decreases in price. Furthermore, the possibility of a coexistence of 'sluggish' and 'non-sluggish' consumers in the market is not mentioned. When products are functionally identical, the presence of 'non-sluggish' consumers (that is, of fully informed consumers without switching costs) precludes the existence of a dynamic price equilibrium in pure strategies, such as the one computed by Selten.

Similarly as in Selten's model, the starting point for this work is the assumption that demand depends upon past market outcomes. The size of a firm's customer base, thus, affects its profitability. The work is divided into three main topics. The first topic are markets with fully informed consumers and switching costs. In such markets, consumers switch suppliers instantaneously (if they switch), but a given price differential does not generally induce all consumers to switch. The second topic are markets with incomplete consumer information (but no switching costs). Here, demand rigidity has a temporal dimension: for a given price differential, market shares change only gradually. The third topic focuses on vertically differentiated markets. Here, demand rigidities play only indirectly a role. It is shown that the standard vertical differentiation model is not able to explain certain empirical observations (see below). Therefore, market imperfections must play a role that are not captured by the standard model. The most plausible explanation is incomplete consumer information, that can lead to rigidities such as the ones described for the second topic. However, they are not explicitly analyzed for vertically differentiated markets.

These three topics are addressed in five chapters. Each chapter focuses on a specific question and can be seen as an independent work. This leads to some redundancies in the introduction of the modeling assumptions, but the advantage is that all chapters are self-contained. In the following, the contents of the five chapters is summarized.

Chapter 1 and 2 focus on the question under what conditions firms can benefit from an increase in the size of their customer base. Chapter 1 analyzes this question in the context of a model with switching costs. For markets with switching costs, it is generally assumed that firms benefit from a large customer base. Klemperer (1995), however, points out that this may not be true if an increase in the size of a firm's customer base induces fiercer price competition, possibly making the firm worse off. Using a duopoly model, chapter 1 shows that such an outcome is not a mere theoretical possibility but can be an empirically relevant result. It is shown to be obtained under plausible conditions, in particular homogeneous goods and dispersed switching costs. It turns out that the relation between the size of a firm's customer base and profit has a maximum at an even *ex-ante* split of the market. This is due to the fact that, under the assumptions of the model, price competition is most intense for a skewed *ex-ante* split of the market. Furthermore, it is shown that the prediction that switching costs tend to make markets less competitive is stronger than previously shown in the literature.

Chapter 2 also deals with the question when firms benefit from an increase in the size of their customer base, but in the context of a model with incomplete consumer information, where consumers are repeat purchasers. Here, the same caveat applies as in chapter 1. A large customer base is not automatically beneficial to a firm if this induces fierce price competition among the competitors. Using a duopoly model where consumers can perform active non-sequential search or word-of-mouth communication, it is shown that the relation between the size of a firm's customer base and profit is not generally monotone and can take on a surprising variety of shapes. When consumers are almost fully informed, a V-shaped relation is obtained. Therefore, a firm is better off with no customer base or with a very large customer base than with an intermediate one. This is due to the fact that price competition is most intense for an even *ex-ante* split of the market. Rigidity that stems from incomplete consumer information, thus, leads to a result that reverses the one obtained for a market with consumer switching costs: there, price competition is least intense for an even *ex-ante* split of the market! In chapter 2, an endogenous determination of the amount of word-of-mouth communication is further introduced, and the model is extended to oligopolies with more than two firms.

The analysis in chapter 3 is based on the same model as the one in chapter 2, but extends it to an infinitely repeated pricing game with rigid demand. The focus is now on the *dynamic* properties of the model. It is a central property of the model that market shares do not converge to a stable split, but fluctuate perpetually. The driving force behind these fluctuations is the non-existence of an equilibrium in pure strategies. Instead, firms randomize over their prices according to an endogenously determined distribution function that depends on the current market split. The resulting *price dispersion* is a well-known property of standard search models.[18] However, until now, authors in this literature paid little attention to the market share dynamics that arise naturally under price dispersion. To the best of my knowledge, the analysis presented in chapter 3, in this respect, novel. It should also be mentioned that the possibility of repeat purchasing plays only a minor role in the existing search literature. In the absence of repeat purchasing, there is – despite incomplete consumer information – of course no demand rigidity. Therefore, the "dynamics" of a standard search model correspond to a simple repetition of a static game. In the model presented here, there is a direct link between the history of the market and the current market outcome. It should also be stressed that, in contrast to a variety of macroeconomic models, the dynamics in the model do not depend on exogenous shocks, but result solely from firms' profit maximizing behavior. The goal of the analysis is to characterize these dynamics, and to reveal the driving forces behind them.

Chapter 4 and 5 focus on vertically differentiated markets. Here, demand rigidities play only indirectly a role. A well-known result for vertically differentiated duopolies with fully informed consumers is that firms differentiate extensively in the quality dimension to soften price competition. This prediction from the theoretical literature has inspired authors to analyze welfare improving policy measures, such as minimum quality standards. Chapter 4 questions the generality of these results by extending the analysis to a larger number of firms.

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[18] The presence of price dispersion in real-world markets has been shown in empirical works. Lach (2002), e.g., writes: "I find that price dispersion prevails even after controlling for observed and unobserved product heterogeneity. Moreover, intra-distribution mobility is significant: stores move up and down the cross-sectional price distribution. Thus, consumers cannot learn about stores that consistently post low prices… price dispersion persists over time as predicted by Varian's (1980) model of sales."

It is shown that the over-differentiation result almost entirely disappears when three or more firms are in the market. Therefore, almost any type of regulation is likely to do more harm than good, since competition already leads to a nearly optimal outcome. A laissez-faire policy should, thus, be adopted. This conclusion is, however, in sharp contrast to what is observed in reality. In particular, minimum quality standards are often used in practice, and their usage is clearly not restricted to duopolistic markets. Under the presumption that minimum quality standards can not generally be interpreted as a sign of over-regulation, it, thus, follows that there must be further sources of market failure that are not captured by the standard model. The most plausible explanation is that consumers can not correctly evaluate the quality of the products in the market. Therefore, the analysis in chapter 4 provides a strong argument for the presence of incomplete consumer information.

In Chapter 5, only the duopoly case of a vertically differentiated market with fully informed consumers is analyzed. It is shown that a widely accepted view, namely that the high-quality provider earns the higher profit (the so-called “high-quality advantage”), is not correct. It is shown that a low-quality advantage can be predicted under perfectly plausible assumptions, such as a concave utility - quality and / or a convex unit cost - quality relation. The existence of a high- or a low-quality advantage depends on the nature of the firms’ strategic interaction for a given specification of the model.

The results of this work suggest that a “profit or market share dilemma”, such as the one that Apple may currently face, can be plausible also from a theoretical standpoint. Under demand rigidity, it can be reasonable for a firm to cut its price or to invest in advertising and, thus, to forego current profit, in order to build up a larger customer base. Depending upon the competitors’ reaction, the larger customer base can be important for the future profitability of the firm.

Chapter 1. ON THE VALUE OF A LARGE CUSTOMER BASE IN MARKETS WITH CONSUMER SWITCHING COSTS

1.1. INTRODUCTION

Casual evidence suggests that managers are sometimes more concerned about market share targets than can be explained by short-run profit maximizing behavior. Apparently, current market shares can determine the future profitability of a firm, which seems rather natural, but can not be explained by standard oligopoly theory (e.g. the Cournot and Bertrand model).

A strand of literature that has resolved this paradox is the one on switching costs. If consumers find it costly to switch from one supplier to another (e.g. because of learning costs of how to use the new product, or transaction costs when opening a new bank account), repeat purchasers are partially locked in. This gives firms some monopoly power that may lead to higher prices in the market. Firms, however, may also compete fiercely for new customers who are not yet locked in. This raises the question how the presence of switching costs affects the overall competitiveness of markets. Most papers in the switching cost literature have focused on this question, and it seems widely accepted that switching costs tend to make markets less competitive.[19]

In contrast to this literature, this chapter focuses on a different question. Namely: To what extend do firms actually have an incentive to build up a large customer base? The answer to this question is crucial when we try to explain why managers are sometimes so concerned about market share targets, and why they are sometimes not. Surprisingly little attention has been paid to this aspect by theorists. Instead, it seems that the idea that firms generally benefit from a large customer base has been taken mostly for granted. Klemperer (1995), however, points out that: "a firm with fewer old customers is relatively more interested in setting a low price to attract new customers than in setting a high price to exploit old customers… this effect can be so strong that a firm can actually be made worse off by increasing its market share, because reducing the competitor's market share makes the competitor so much more aggressive".

Although the idea that a firm can be made worse off by an increase in the size of its customer base is not new, there remains the flavor of a mere theoretical possibility with little empirical relevance, except for a few special cases. One of these special cases is illustrated in

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[19] See Klemperer, 1987b and 1995; Beggs and Klemperer, 1992; Padilla, 1995.

Klemperer (1987c), where an incumbent firm faces a potential entrant in a Cournot market with switching costs. If the incumbent firm's customer base reaches a critical size, entry becomes profitable, which leads to a discontinuous drop in the incumbent's profit. The incumbent may, thus, find it profitable to disinvest in the size of its customer base in order to deter entry.

This chapter shows that a non-monotonic relation between the size of a firm's customer base and profit can be an empirically relevant prediction in the absence of a potential entrant. It is obtained under simple and for some markets plausible conditions. In particular, I assume that products in a duopolistic market are homogeneous, and that consumers have dispersed switching costs[20]. This is sufficient to generate a non-monotonic relation between the size of a firm's customer base and profit. It is shown that profits are maximized for an even *ex-ante* split of the market, that is, when each firm served half of the market in the past. The reason for this is, that under the assumptions of the model, price competition is more intense for skewed splits of the market, and softer for even splits. The effect is so strong that a firm does not have an incentive to increase the size of its customer base beyond 50 percent.

If firms do not benefit from an increase in the size of their customer base, they do not have an incentive to invest in it. This has important dynamic implications, in particular for earlier stages of the market, where consumers are not yet attached to one of the suppliers. Previous works on markets with switching costs have shown that competition is intense in earlier market stages (see e.g. Klemperer, 1987a). In this chapter, however, firms do not benefit from an increase in the size of their customer base beyond 50 percent. Therefore, it is no longer obvious that competition should be particularly intense in earlier market stages. In order to analyze how the above non-monotonicity result affects competition in early market stages, I added a first stage to the game where consumers are not yet attached to firms. It is shown that a continuum of equilibrium prices can be sustained in the first stage. Assuming that firms always coordinate on the highest equilibrium price, the unambiguous prediction of the model is that switching costs lead to higher prices in both periods than in the absence of switching costs. This prediction is stronger than previously shown in the literature, where, depending on the parameter values, switching costs can lead to lower prices in earlier stages of the market than without switching costs (see Klemperer, 1987b).

In Section 1.2, the model is introduced and the results are shown. Section 1.3 concludes.

[20] Many authors assume that switching costs are identical for all consumers (e.g. Klemperer, 1987b; Padilla, 1995).

1.2. MODEL & RESULTS

Consider a market with a continuum of consumers with measure one. There are two periods, and each marginal consumer buys either zero or one unit of a homogeneous product in each period. Consumers' reservation prices are represented by some distribution function $G(r)$, with support $[\underline{r},\overline{r}]$. Demand is served by two firms ($i=1,2$) that produce with zero marginal costs. If a consumer purchases a different product in the second period than in the first, a switching cost is incurred. Consumers' switching costs are represented by the distribution function $F(s)$, with support $[\underline{s},\overline{s}]$. Consumers discover their individual switching cost only after consuming a product in this market. Therefore, consumers are *ex-ante* identical and do not make their first-period choice contingent on the level of their switching cost.

Prices in period 1 are denoted by q_1 and q_2, and prices in period 2 by p_1 and p_2, respectively. Let the size of firm 1's *customer base* at the beginning of period 2 be firm 1's demand in period 1, and denote it by n. If all consumers make a purchase, firm 2's demand in period 1 is, thus, $1-n$. Firm 1's and firm 2's demand in period 2 are denoted by D_1 and D_2, respectively. The analysis in this chapter focuses on situations where switching costs are small relative to reservation prices. Most of the probability mass in the distribution function $G(r)$ is, thus, at prices above $\overline{s}$. The easiest way to formalize this is to assume that $\underline{r} > \overline{s}$. It can be shown that prices above $\overline{s}$ are not chosen in equilibrium (see below). Therefore, in both periods, aggregate demand equals 1, and results are independent of the specification of $G(r)$.[21] Furthermore, prices are normalized such that $\overline{s} \equiv 1$.

Under the above assumptions, firm 1's demand in period 2 is given by:[22]

$$D_1(p_1 \mid p_2) = n - nF(p_1 - p_2) + (1-n)F(p_2 - p_1) \tag{1}$$

Firm 2's demand in period 2 equals $1-D_1$. Period-2-profits are given by $\pi_i = p_i D_i$, $i=1,2$.

To obtain simple analytical results, I will from now on assume that switching costs are uniformly distributed, and that $\underline{s} \equiv 0$. Therefore:

$$F(s) = \begin{cases} 1 & , s > 1 \\ s & , 0 \le s \le 1 \\ 0 & , s < 0 \end{cases} \tag{2}$$

Using (1) and (2), we obtain the following expression for firm 1's profit in period 2:

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[21] The analysis includes the special case where all consumers have an identical reservation price.

[22] Note, that $F(s)=0$ for $s < \underline{s}$, and $F(s)=1$ for $s > \overline{s}$.

$$\pi_1(p_1 \mid p_2) = p_1 \cdot \begin{cases} 0 & , p_1 > p_2 + 1 \\ n(1 - p_1 + p_2) & , p_2 \le p_1 \le p_2 + 1 \\ n + (1-n)(p_2 - p_1) & , p_2 - 1 \le p_1 < p_2 \\ 1 & , p_1 < p_2 - 1 \end{cases} \tag{3}$$

, and similarly for firm 2.

(3) implies that in the maximization of π_1 over p_1, firm 1 never chooses a price outside the interval $[p_2 - 1, p_2 + 1]$. For the interior of this interval, firm 1's best reply function must be computed separately for the case $p_1 < p_2$ and $p_1 > p_2$ (similarly for firm 2). The two firms' best reply functions intersect only once in the relevant range. For $n \ge 1/2$, this intersection point is given by:

$$p_1^{n\ge 0.5} = \frac{1+n}{3n},\ p_2^{n\ge 0.5} = \frac{2-n}{3n} \tag{4}$$

In this candidate equilibrium, firm 2 (the firm with the smaller customer base) is the low-priced firm. To show that (4) is an equilibrium, we must verify that firm 2 does not benefit from deviating to become the high-priced firm.[23] If firm 2 deviates from the candidate equilibrium in (4) to become the high-priced firm, it earns the following profit (using (3), and firm 2's best reply for $p_2 > p_1$):

$$\pi_2^{dev}(n) = \frac{1-n}{4}\left(\frac{4n+1}{3n}\right)^2 \tag{5}$$

(4) is an equilibrium if firm 2's profit is not smaller than $\pi_2^{dev}(n)$ for any $n \in [0.5,1]$. Firm 2's profit in the candidate equilibrium (4) equals (using (3)):

$$\pi_2^{n\ge 0.5}(n) = \frac{(2-n)^2}{9n} \tag{6}$$

It is easy to verify that $\pi_2^{n\ge 0.5}(n) \ge \pi_2^{dev}(n)$ holds $\forall n \in [0.5,1]$, so (4) is an equilibrium.

Analog results are obtained for $n < 1/2$, but the roles of the two firms as high-priced and low-priced firm are reversed. One obtains for firm 1's price over the entire range of n:

$$p_1^*(n) = \begin{cases} (1+n)/3n & , n \ge 0.5 \\ (1+n)/3(1-n) & , n < 0.5 \end{cases} \tag{7}$$

[23] Formally, firm 2's best reply function is discontinuous. There is a critical price p_1 between 0 and 1 where firm 2 "switches" from being the high-priced to being the low-priced firm. We must verify that this switching point lies to the left of the candidate equilibrium price $p_1^{n\ge 0.5}$ in (4).

Plotting p_1^* over n using (7), we find that firm 1's equilibrium price increases in n for $n<1/2$, and decreases in n for $n>1/2$. The maximum price of $p_1=1$ is attained at $n=1/2$.[24] Therefore, price competition in period 2 is most intense for skewed *ex-ante* splits of the market, and least intense for even splits. This suggests that a firm that has served at least half of the market in the past, may not benefit from an increase in the size of its customer base (in terms of period-2-profit) if the intensified price competition outweighs the benefit a large customer base has *per se* (at fixed prices). To see whether this is the case, we must analyze firm 1's (or equivalently firm 2's) profit as a function of n.

Using (3) and (7), we obtain the following expression for firm 1's period-2-profit:

$$\pi_1^*(n)=\begin{cases}(1+n)^2/9n & ,\ n\geq 0.5\\ (1+n)^2/9(1-n) & ,\ n<0.5\end{cases} \tag{8}$$

Figure 1 shows π_1 as a function of n.[25]

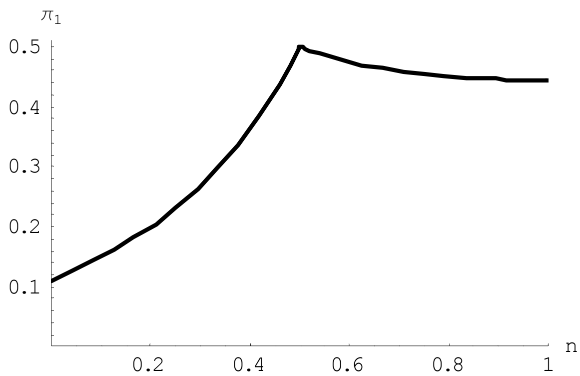

Figure 1: Firm 1's period-2-profit $\pi_1$as a function of its customer base size n

The main result is summarized in the following Proposition:

Proposition 1: Firm 1's second-period profit increases in the size of its customer base n if $n<1/2$, and it decreases in n if $n>1/2$.

According to Proposition 1, firm 1 does not benefit from an increase in the size of its customer base (in terms of period-2-profit) if it has already served at least half of the market

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[24] Prices above 1 are, thus, never chosen in equilibrium. Note, that a deviation to a price above $\underline{r}$, the lowest reservation price, can never be profitable as this leads to a double loss of customers: 1. of customers who switch to the competitor, and 2. of customers who neither want to switch, nor make a purchase at the given price.

[25] The analog curve for firm 2 is just the mirror picture and contains no additional information.

in the previous period. This result contradicts the commonly held belief that firms generally benefit from a large customer base in markets with switching costs. It is important to note that the non-monotonic relation between the size of firm 1's customer base and firm 1's period-2-profit was obtained using simple and for some markets plausible assumptions, in particular homogeneous goods and uniformly distributed switching costs. The result should, thus, not be seen as a theoretical curiosity, but as an empirically relevant prediction.[26]

In the following, I will investigate how the above findings affect competition in earlier stages of the market, when consumers are not yet attached to one of the suppliers. Previous works have shown that competition in markets with switching costs may be particularly intense in early market stages because firms have an incentive to build up a large customer base that is valuable at later stages. According to the above result, however, this may not generally be true. In the present model, firms have an incentive to build up a customer base, but only as long as its size does not exceed 1/2. Therefore, it is no longer obvious that competition should be particularly intense in period 1.

Before moving on to the fully rational case, a useful benchmark is to study the case of myopic consumers who do not take into account how their first-period choice affects their second-period options.

Case 1: Myopic consumers

Myopic consumers buy from the cheaper supplier in period 1. Therefore, if $q_1 < q_2$, then $n = 1$. If $q_1 > q_2$, $n = 0$. If $q_1 = q_2$, consumers are assumed to randomize, so $n = 1/2$. Using (8), we obtain for firm 1's period-2-profits in these cases: $\pi_1(1) = 4/9$, $\pi_1(0) = 1/9$, and $\pi_1(1/2) = 1/2$. If future profits are discounted with the factor δ, we obtain for firm 1's total discounted profit in period 1:

$$\Pi_1(q_1 \mid q_2) = \begin{cases} \delta/9 & , \ q_1 > q_2 \\ (q_1 + \delta)/2 & , \ q_1 = q_2 \\ q_1 + \frac{4}{9}\delta & , \ q_1 < q_2 \end{cases} \tag{9}$$

[26] Non-monotonic relations between the size of a firm' customer base and profit must not always be single-peaked as in Figure 1. The type of non-monotonicity obtained depends on when price competition is particularly intense. See chapter 2.

Proposition 2: There is a continuum of equilibria. The set of equilibrium prices $(q_1^*, q_2^*, p_1^*, p_2^*)$ is given by: $\left\{(q,q,p,p) \mid q \in \left[-\tfrac{7}{9}\delta, \tfrac{1}{9}\delta\right], p=1\right\}$.

Proof:

If both firms charge an identical price q in period 1, then the equilibrium price of $p^* = 1$ in period 2 follows immediately from (4). To charge an identical price in period 1 is an equilibrium outcome if no deviation is profitable. Undercutting q marginally yields a profit of $q + \tfrac{4}{9}\delta$. Equalizing this with the profit from not deviating, $(q+\delta)/2$, yields for the highest sustainable equilibrium price in period 1: $q^{\max} = \tfrac{1}{9}\delta$. Choosing a higher price than q yields a profit of $\delta/9$. Equalizing this with $(q+\delta)/2$, we obtain for the lowest sustainable price in period 1: $q^{\min} = -\tfrac{7}{9}\delta$. Outside the interval $[q^{\min}, q^{\max}]$, a profitable deviation exists, inside, there is no profitable deviation. There are no asymmetric equilibria because, as can easily be verified, at least one of the firms always has an incentive to deviate. □

According to Proposition 2, the highest sustainable price in period 1 is $q^{\max} = \delta/9$. Assuming that firms can coordinate on this price, we obtain the equilibrium profits:

$$\Pi_1^* = \Pi_2^* = \frac{5}{9}\delta \tag{10}$$

Similarly as in a collusive setting, firms can achieve a higher profit for larger values of the discount factor δ. Note, that this is not only due to the higher valuation of second-period profits in period 1. According to $q^{\max} = \delta/9$, firms can also coordinate on higher first-period prices, the more valuable second-period profits are! This reverses the basic intuition, according to which firms, in an attempt to build up a large customer base, compete more fiercely for first-period sales the more valuable future profits are. For $\delta \to 0$, the first-period outcome converges to the competitive one.

Case 2: Rational consumers

In period 1, rational consumers observe q_1 and q_2 and anticipate second-period prices p_1 and p_2 (as functions of n). Unless there is an equilibrium where all consumers purchase from the same supplier in period 1, the equilibrium value of n will be such that consumers are indifferent between firm 1's and firm 2's offer. Otherwise, consumers would deviate from their period-1 choice and purchase from the other supplier.

Lemma 1: There is no equilibrium where all consumers purchase from the same firm in period 1.

Proof:

Suppose to the contrary that there is an equilibrium where all consumers purchase from, say, firm 1 in period 1 ($n^* = 1$). There are two cases to be distinguished: 1. $q_1^* \geq 0$, and 2. $q_1^* < 0$. Consider the first case. Firm 2 has an incentive to deviate from $q_2^* > q_1^*$ and match q_1^*. This yields a non-negative profit in period 1, and, by Proposition 1, unambiguously leads to an increase in its second-period profit. Now consider the second case. Firm 1 makes losses in period 1. Therefore, it has an incentive to increase its price and match q_2^*. This reduces firm 1's first-period losses, and unambiguously leads to an increase in its second-period profit.

As a consequence of Lemma 1, we only have to look for equilibria where consumers are indifferent between firm 1's and firm 2's offer in period 1. For given prices q_1 and q_2, n is, thus, determined by a condition of indifference. The expected discounted expenditure of a marginal consumer who purchases from firm 1 in period 1 is (I assume that consumers discount the future with the same rate as firms):

$$q_1 + \delta \Pr[1 \to 1] p_1 + \delta \Pr[1 \to 2]\left(p_2 + E[s \mid 1 \to 2]\right) \tag{11}$$

, where "$1 \to 2$", e.g., denotes the event that the consumer purchases from firm 1 in period 1, and switches to firm 2 in period 2. An analog expression is obtained for firm 2. In equilibrium, the two expressions must be equal. Consumers are, then, indifferent between the two suppliers.

A consumer switches from firm 1 to firm 2 in period 2 if her cost of switching is smaller than the price differential $p_1 - p_2$. Therefore, we have: $\Pr[1 \to 2] = F(p_1 - p_2)$, and $\Pr[1 \to 1] = 1 - F(p_1 - p_2)$. Similarly, $\Pr[2 \to 1] = F(p_2 - p_1)$ and $\Pr[2 \to 2] = 1 - F(p_2 - p_1)$. Assuming that $q_1 \leq q_2$ without loss of generality, it follows that $n \geq 1/2$ and $p_1 \geq p_2$ (otherwise, firm 1 would be the low-priced firm in both periods, so all consumers would buy from firm 1 in period 1). Using (2), we, thus, obtain: $\Pr[1 \to 2] = p_1 - p_2$, $\Pr[1 \to 1] = 1 - p_1 + p_2$, $\Pr[2 \to 1] = 0$, and $\Pr[2 \to 2] = 1$. For the conditional expectation in

(11), we get: $E[s\,|\,1\to 2]=(p_1-p_2)/2$. This yields the following condition of indifference that implicitly defines n:

$$q_1+\delta p_1(1-p_1+p_2)+\delta(p_1-p_2)\big(p_2+(p_1-p_2)/2\big)=q_2+\delta p_2 \tag{12}$$

Consumers correctly anticipate second-period prices. Therefore, we can use (4) to replace p_1 and p_2 in (12). Solving for n, we, thus, obtain the following dependency of first-period market shares from first-period prices:

$$n(q_1,q_2)=\frac{\delta+3\sqrt{\delta\big(\delta-2(q_2-q_1)\big)}}{8\delta-18(q_2-q_1)} \tag{13}$$

An analog expression is obtained for $q_1>q_2$.

For a given value of q_2, firm 1 maximizes:

$$\Pi_1(q_1\,|\,q_2)=\begin{cases} q_1 n(q_1,q_2)+\delta\dfrac{\big(1+n(q_1,q_2)\big)^2}{9n(q_1,q_2)} & , \; q_1\le q_2 \\ q_1 n(q_1,q_2)+\delta\dfrac{\big(1+n(q_1,q_2)\big)^2}{9\big(1-n(q_1,q_2)\big)} & , \; q_1>q_2 \end{cases} \tag{14}$$

It can be shown that firm 1 never benefits from choosing a price greater than q_2 ($\Pi_1(q_1\,|\,q_2)$ is decreasing in q_1 for $q_1>q_2\ge 0$). The same is true for firm 2. Therefore, if a pure strategy equilibrium exists, then it must be symmetric: $q_1^*=q_2^*\equiv q$. The resulting profit is:

$$\Pi_1^*=\Pi_2^*=(q+\delta)/2 \tag{15}$$

A pure strategy equilibrium fails to exist if for a given value for $q_2=q$, $q_1=q$ is not a global maximum of (14). It turns out that this is the case if q is relatively large. A deviation to a price below q is, then, profitable. This is intuitive, because firm 1 is obviously more interested in undercutting $q_2=q$ when q_2 is large (note, that $n(q_1,q_2)$ depends only on the difference between q_1 and q_2). On the other hand, when q is small, undercutting q yields little or no additional profit in period 1, whereas profit in period 2 decreases according to Proposition 1. Therefore, as in the case with myopic consumers, there is an upper bound $q^{\max}$ to the set of first-period prices that can be sustained in equilibrium.

To compute $q^{\max}$, we have to know which deviation from q is most profitable (if any). $q^{\max}$ is, then, the highest common price level q from which no profitable deviation exists. It can be shown that, for intermediate values of q, $\Pi_1(q_1\,|\,q_2=q)$ is U-shaped for $q_1\le q$. Therefore, the most profitable deviation is to choose the highest price that is sufficiently low to attract the entire market demand in period 1 ($n=1$). Using (13), we obtain for this price:

$$q^{dev} = q - \frac{5}{18}\delta \tag{16}$$

The highest sustainable equilibrium price in period 1 is, thus, determined by the condition:

$$\Pi_1(q_1 = q_2 = q^{\max}) = \Pi_1(q_1 = q^{dev}, q_2 = q^{\max}) \tag{17}$$

Using (14), this yields:

$$q^{\max} = \frac{2}{3}\delta \tag{18}$$

The lowest sustainable price $q^{\min}$ is computed in a similar way. Firm 1's best deviation upwards when q is negative is to choose a price q_1 that is sufficiently high so that $n=0$. Therefore, firm 1's first-period profit equals zero, and the second-period profit equals $\pi_1(0)=1/9$. One obtains the same value for the lowest sustainable price as in the case with myopic consumers: $q^{\min} = -\frac{7}{9}\delta$.

It is plausible to assume that firms coordinate on the highest sustainable price in period 1. This yields for the equilibrium profits:

$$\Pi_1^* = \Pi_2^* = \frac{5}{6}\delta \tag{19}$$

The results are summarized by the following Proposition:

Proposition 3: When consumers are rational and forward-looking, first-period prices are above the competitive price, and they are larger than in the case with myopic consumers. In both cases, first-period prices are increasing in the discount factor δ.

That first-period prices are higher when consumers are rational than in the case with myopic consumers is not surprising. Myopic consumers purchase from the cheaper supplier in period 1. This makes price competition fairly intense. When consumers are rational and forward-looking, they realize that a firm that cuts its price in period 1 will charge a higher price in period 2 than its competitor. This prevents that all consumers pick the cheap supplier in period 1, and demand becomes continuous in prices. Firms' incentives to undercut the competitor's price are, thus, lower, and equilibrium prices higher than in the myopic case.

The most fundamental insight from the analysis is, that the non-monotonicity in the relation between the size of a firm's customer base and profit in period 2 allows firms to sustain prices above the competitive level also in period 1. Unlike in previous works, price competition in earlier stages of a market with switching costs is, thus, not particularly intense (in the present model, it is only slightly more intense in the first than in the second period). This is because firms have only a limited interest in building up a customer base. A large customer base is less valuable than an intermediate one because it implies tough price competition in the second period. The strength of this effect depends upon the valuation of future profits. If they are fully discounted ($\delta = 0$), first-period prices collapse to the competitive level. The finding that a higher discount factor yields higher profits is reminiscent of a collusive outcome, but it is entirely based on switching costs.

1.3. CONCLUSION

It is usually assumed than, in markets where current market shares determine the future profitability of firms, firms generally benefit from a large customer base. The present chapter shows that this is not true for markets where consumers face costs of switching the supplier. A non-monotonic relation between the size of a firm's customer base and profit is obtained under plausible conditions, in particular homogeneous goods and dispersed switching cost. Under the assumptions of the model, price competition is least intense for an even *ex-ante* split of the market. The effect is so strong that firms do not benefit from an increase in the size of their customer base beyond 50 percent. This prediction is in sharp contrast with the existing literature on switching costs, according to which firms generally have an incentive to *invest* in future market shares. In fact, the entire literature on dynamic price competition with switching costs is based on this idea.[27] If firms have only a limited interest in building up a customer base, there are fewer incentives to compete for consumers who are not yet attached to one of the suppliers in the market. This strengthens the notion that switching costs lead to higher prices in the market than in the absence of switching costs.

[27] see, e.g., Klemperer (1987) and Padilla (1995)

Chapter 2. On the Value of a Large Customer Base in Markets with Incomplete Consumer Information

2.1. Introduction

Firms can only compete for consumers who are aware of the existence and the prices of their products, as well as about the products' main characteristics and the suppliers' locations. Therefore, the competitiveness of product markets depends crucially upon the amount of information available to the consumers. Many authors abstract from informational constraints and assume full information, without specifying the sources of the relevant information. Such sources are, e.g., advertising, active search, or consumers' word-of-mouth communication.[28] The latter has received little attention in the economics literature, although casual evidence suggests that it is an important source of information.[29] In markets with repeat purchasing, a major source of information are also the consumers' own previous consumption experiences. Consumers may, e.g., remember the location of their previous supplier and the product's characteristics. If consumers have superior information about their previous supplier, a direct link between past sales and current demand emerges.

In this chapter, I analyze price competition in a market characterized by incomplete consumer information and repeat purchasing. Consumers gather information via active search or via word-of-mouth communication. In the model, there is a direct link between a firm's previous sales and current demand. The main question that shall be answered is whether it is generally true that firms benefit from having a large customer base, as economic intuition would suggest. Although a static modeling framework is chosen, the inclusion of firms' past market shares introduces an important dynamic aspect into the model.

Apart from the word-of-mouth information process, the main difference between the work in this chapter and most of the existing literature (e.g. on consumer search) is the explicit treatment of *ex-ante* asymmetric splits of the market, and the analysis of the dependency of current profits from past sales. I have not found any detailed study on the shape of past sales – current profit relations anywhere in the literature. This chapter performs this exercise for markets characterized by incomplete consumer information and repeat purchasing.

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[28] Another way to gather information is to observe other consumers' choices.

[29] Word-of-mouth seems especially prevalent in markets for music, film, literature, restaurants etc..

The main assumptions of the model are as follows. There is a homogeneous good duopoly, and a unit mass of consumers with a uniform reservation price. The market is characterized by repeat purchasing, and all consumers have made a purchase before. The consumers who purchased a firm's product in the past are the firm's *customer base*. The size of a firm's customer base matters because consumers are assumed to have information from past consumption experiences. In particular, consumers are aware of existence and location of their previous supplier, and about the respective product's characteristics. Given this information, a current price quote can always be obtained for free.

There are three types of consumers. The first type uses word-of-mouth as a potential source of information about the other supplier. Each 'word-of-mouth consumer' asks one other consumer randomly chosen from the population about her previous choice. If it differs from the consumer's own previous choice, the relevant information on the other supplier's existence, location, and product characteristics[30] is transmitted, and a current price quote is obtained for free (see Butters, 1977, and Fishman and Rob, 1995). Consumers who are aware of both firms' offers always buy from the firm that currently offers the lower price.[31] Word-of-mouth consumers who ask consumers from their own customer base do not find out about the other firm's offer and buy from their previous supplier unless the current price exceeds the reservation price. Since the probability that a word-of-mouth consumer asks a consumer from a given firm's customer base increases in the size of this customer base, a firm that previously served a large fraction of market demand is in an advantageous position. The word-of-mouth process, thus, exhibits "popularity weighting". The second type of consumers are the "searchers" who perform active non-sequential search and are always informed about both firms' offers. The third consumer type does not search actively or communicate via word-of-mouth. Consumers of this "ignorant type" simply return to their previous supplier. Other authors, e.g. Salop and Stiglitz (1977), Varian (1980), Janssen and Moraga-Gonzales (2004), also introduced consumers who learn only one supplier's price, but assume that they choose the supplier *randomly*. The above assumptions about the ignorant types and the word-of-mouth consumers formalize the idea that consumers are repeat purchasers with superior information about their previous supplier.

It is shown that no pure strategy equilibrium exists, and an equilibrium in mixed pricing strategies is derived. Market outcomes are, thus, characterized by price distributions, a property well-known from the search literature. The model allows for asymmetric

[30] Although products are assumed to be homogeneous in the model, consumers may not be aware of this.

[31] If both firms offer the same price, a tie braking rule is applied.

distributions of past market shares and for non-identical marginal production costs. It offers not only a resolution to the “Bertrand Paradox”, according to which under full information, no capacity constraints, and identical marginal costs, firms price competitively. It also offers a resolution to what might be referred to as a “second Bertrand Paradox”, which states that whenever there is a difference in marginal production costs, a high-cost firm obtains zero market share. In this chapter, the inefficient firm retains a positive market share if it has a positive customer base. If it does not have a customer base (entry), the incumbent conducts limit pricing if the entrant’s marginal costs are sufficiently higher than the incumbent’s.

The central question that shall be answered is whether it is generally true that current (expected) profit monotonically increases in the size of a firm’s customer base. It turns out that this is not the case. There is, in fact, a surprising variety of possible shapes of this relation. E.g., when most consumers are searchers, so we are only a small step away from the standard Bertrand model with full information, the past sales – current profit relation is V-shaped. The reason for this result is that price competition is particularly intense for an even *ex-ante* split of the market.[32] A reduction in the size of a firm’s customer base is, thus, in the firm’s interest if the benefit of softened price competition outweighs the loss of monopoly power over uninformed repeat purchasers.

In most sections of this chapter, the fraction of word-of-mouth consumers and ignorant types in the population is treated as exogenous. The rationale behind this is that consumers are often poorly informed about the characteristics of markets, such as the number of suppliers, technology, aggregate demand etc.. Therefore, they can not condition their search decision upon these characteristics.[33] An alternative explanation is that searchers enjoy shopping and have negligible search costs, word-of-mouth consumers enjoy chatting, while the ignorant types find both activities prohibitively costly. In order to analyze under what conditions well informed rational consumers rely on word-of-mouth, and to check the robustness of the main non-monotonicity result, I introduce an endogenization of the fraction of word-of-mouth consumers in the population. To this end, I assume that active search is costly, and word-of-mouth communication is for free (as a by-product of social interaction). Consumers are

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[32] In chapter 1, I showed that in a duopolistic market with full information and switching costs, profits are maximized for an *even ex-ante* split of the market. This finding is in stark contrast to the V-shaped past sales – current profit relation derived in this chapter.

[33] Hehenkamp (2002) introduces an evolutionary model where both consumers and firms are poorly informed, and firms make decisions by imitation and experimentation. However, it seems plausible to assume that consumers (who are active in a variety of markets) are poorly informed about a specific market, while firms (that are active in few markets) are well informed. This view is adopted in the present chapter.

informed about the main characteristics of the market, including the past distribution of market shares, and they use this information to compute expected utility gains or losses from active search. It is shown that the equilibrium fraction of word-of-mouth consumers is (weakly) increasing in the search cost. For some parameter values, there are multiple equilibria: an equilibrium with a large fraction of searchers, and a "no-search-equilibrium" where all consumers rely on word-of-mouth.

Finally, the analysis is extended to a larger number of firms. The main question is whether the non-monotonicities that were observed in the duopoly case persist when the number of competitors increases. It is shown that they can persist if the number of firms remains finite, but they always disappear when the number of firms approaches infinity. I further analyze whether prices and aggregate industry profits converge to zero as the number of firms approaches infinity, a question that has often been analyzed in the search literature.

It is important to note, that a *static* modeling framework is chosen in this chapter, so firms' incentives to invest (or disinvest) in future market shares are excluded. Although a full-fledged dynamic analysis would be desirable, the static approach chosen in the present chapter is justified for two reasons. First, a dynamic analysis is only useful when the properties of the static game are already well understood since the effects that are found in the static game are also present in a dynamic setting. This understanding is provided in this chapter, and it is shown that the static game is rich enough to deserve special attention in its own right. And more importantly, the static game is shown to be a good approximation of a more general dynamic model under certain conditions (see Section 2.6). In brief, suppose the products sold in our market are durable, but durability is finite. Therefore, only a fraction of all potential consumers make a purchase each period, an assumption that holds for many real-world markets.[34] As a consequence, even if a firm serves a large fraction of current market demand, the total size of its customer base (including all consumers who purchased the firm's product when they last made a purchase) remains small if it served only a small fraction of market demand in previous periods. It is shown that, as the durability of the products becomes large, the initial split of the market next period (the state) becomes insensitive to current prices. The static model is, thus, obtained in the limit as durability approaches infinity.

A brief review of the related literature:

The model is related to the existing literature on word-of-mouth communication. Ellison and Fudenberg (1995), e.g., analyze the effectiveness of word-of-mouth communication in a non-

[34] Word-of-mouth consumers are, thus, likely to ask consumers who's last purchase is several periods ago.

market situation. They consider a population in which each individual can choose between two alternatives. Payoffs are stochastic with an idiosyncratic and a common component. Agents who re-evaluate their last-period choice ask a sample of N other agents about their last-period choice and payoff.[35] The other agents stick to their previous choice. The authors show that there is convergence towards the alternative that is on average superior if the sample size (N) is not too large.

In this chapter, it is assumed that word-of-mouth consumers ask only *one* other consumer instead of $N \geq 1$. Although this formalization is less general, I believe that it captures the essence of word-of-mouth communication (in particular the 'popularity weighting' property) sufficiently well while achieving analytical tractability. Consumers who ask several other consumers are well informed and, thus, similar to searchers. The searchers in this chapter may be interpreted as word-of-mouth consumers with a larger sample size.

Rob and Fishman (2005) introduce a dynamic model of a product market with continuous investments in quality. In their model, word-of-mouth communication is defined as follows. Consumers live for one period. New consumers who enter the market meet with a certain probability old consumers who leave the market and find out about the "tenure" of the firm that was patronized by the old consumer. The 'tenure' is defined as the number of periods since the firm last produced a low-quality product. Each new consumer can either visit the firm that she finds out about via word-of-mouth (if any), or search for another firm, where to 'search' means simply to choose another supplier randomly.[36] In contrast to the model introduced in this chapter, the authors abstract from price competition by assuming that each supplier has monopoly power over the consumers visiting it this period. The main result is that firms' incentives to invest in quality are increasing in the size of their customer base.[37]

The present model is also closely related to the literature on consumer search. Stiglitz (1989), e.g., analyzes search in a market where consumers find out about the price of a randomly chosen store without cost. To obtain a price quote from a second store, a search cost is incurred. More search is prohibitively costly. The author shows that, when the number of

[35] Satterthwaite (1979) introduces a word-of-mouth process where consumers pass on information also about suppliers they have heard of from other consumers, and gradually forget the information over time.

[36] This word-of-mouth process could also be interpreted in terms of repeat-purchasing, assuming that only a certain fraction of the consumers leaves the market each period, and those who stay are aware of the tenure of their previous supplier.

[37] Vettas (1997) considers a monopolist who sells a durable good in a market where consumers communicate via word-of-mouth. Consumers who purchase the good become informed about its quality and pass on this information to other consumers. Therefore, the quantity sold affects consumers' information about quality.

stores is raised, the average price increases. Stiglitz argues that word-of-mouth communication (with "friendly neighbors") can be modeled in the same way if one assumes that each consumer visits one randomly chosen store, and some consumers find out about the price of a second store by talking to a neighbor. Stahl (1989) analyzes sequential search in a market where an exogenous fraction of the consumers (the "shoppers") have zero search cost. The others incur a cost whenever they visit a different store, but the first search is free.

Similarly as Stiglitz (1989) and Stahl (1989), many authors of search models assume that consumers can obtain the first price quote for free. Since they do not possess knowledge from past consumption experiences, consumers *randomize* over the identity of the first store they visit.[38] Alternatively, one could assume there is repeat purchasing, and the store from which consumers obtain the free price quote is the one where they made their last purchase. But since all authors have restricted their attention to *ex-ante symmetric* situations, this interpretation would imply that all firms had identical market shares in the past. The main contribution of this chapter is to consider *ex-ante asymmetric* situations and to perform an explicit analysis of the past sales – current profit relation.

The rest of this chapter is organized as follows. In Section 2.2, the model is introduced, and the main properties of the equilibrium are identified. The analysis is performed on a general level and allows for other information processes than the one introduced in this chapter. In Section 2.3, the results are applied to the word-of-mouth information process, and the shape of the past sales – current profit relation is analyzed. Section 2.4 contains the endogenization of the fraction of word-of-mouth consumers. Section 2.5 extends the analysis to oligopolies with more than two firms. Section 2.6 contains a discussion of the main modeling choices. Section 2.7 concludes. All proofs are relegated to the Appendix.

2.2. THE MODEL

Consider a homogeneous good duopoly where firms produce with constant marginal cost technologies, and let $c_i < 1$ ($i \in \{1,2\}$) denote firm i's marginal cost of production. On the demand side, there is a continuum of consumers of measure one. Each (marginal) consumer

[38] Bester and Petrakis (1995) assume that consumers live in the neighborhood of either store 1 or store 2. The only way to find out about the other store's price is via advertising. The authors assume that a transportation cost is incurred when the distant store is visited.

buys either zero or one unit in the market. All consumers have the same reservation price, normalized to 1 without loss of generality.[39] Note, that the monopoly price also equals 1.

Suppose, there is repeat purchasing, and the market is 'mature' in the sense that all consumers have made a purchase in this market before. Let the size of firm i's *customer base*, n_i, be the mass of consumers who purchased firm i's product when they last made a purchase. Therefore, n_i is equal to firm i's previous market share.

By assumption, consumers are not fully informed about the existence or the location of the suppliers or about the relevant characteristics of the products. Therefore, the firm that charges the lower price in the market will generally not serve the entire market demand. Let $h(n)$ be the demand of the low-priced firm when the size of its customer base is n. Similarly, let $l(n)$ be the firm's demand when it charges the higher price. I assume that firms are identical except for the size of their customer base and their marginal production costs. Therefore, the functions $l(.)$ and $h(.)$ do not depend on the identity of the firm ($i \in \{1,2\}$). A specification of $l(.)$ and $h(.)$ is called an *information process*.

Without loss of generality, strategies with $p_i > 1$ are eliminated from the strategy space. Firm i's demand is, thus, given by ($i, j \in \{1,2\}$, $j \neq i$):

$$D_i(p_i \mid p_j) = \begin{cases} h(n_i) & \text{if } p_i \in [0, p_j) \\ n_i & \text{if } p_i = p_j \\ l(n_i) & \text{if } p_i \in (p_j, 1] \end{cases} \tag{1}$$

Firm i's *ex-post* profit equals $\pi_i(p_i \mid p_j) = (p_i - c_i) D_i(p_i \mid p_j)$.

Under the above assumptions, the following *symmetry property* holds generally for all information processes $\left(l(.), h(.)\right)$ and $\forall n \in [0,1]$:

$$l(n) = 1 - h(1-n) \tag{2}$$

This follows immediately from $l(n_i) + h(n_j) = 1$ if $p_i > p_j$ and if $p_i < p_j$ (all consumers buy), and from the 'mature market' condition: $n_i + n_j = 1$.

Definition 1: A symmetric information process $\left(l(.), h(.)\right)$ is *strictly monotone* if $h(.)$ is strictly increasing. It is *imperfect* if $h(n) < 1$ and *non-trivial* if $h(n) > n \quad \forall n \in [0,1)$. An

[39] The uniform reservation price could be replaced by a downward-sloping demand function, but this would add little to our understanding of the effects that drive the main results in this chapter.

information process $\left(l(.), h(.)\right)$ that is symmetric, strictly monotone, imperfect, and non-trivial is referred to as a "SMIT" information process.

Lemma 1: Given some SMIT information process, if both firms have a positive[40] customer base, no equilibrium in pure strategies exists.

According to Lemma 1, if an equilibrium exists, then market outcomes are characterized by *price distributions*. Pure strategy equilibria are only obtained if one of the conditions in Lemma 1 is violated. E.g., if the information process is not imperfect, the standard Bertrand model with full information is obtained. If the information process is trivial, both firms choose the monopoly price with probability one. If one of the firms had a market share of size zero in the past (this case can be used to describe entry), and this firm's marginal cost is sufficiently higher than the incumbent's, then the incumbent conducts limit pricing (see below). The mixed strategy equilibrium derived in this section is well-behaved in the sense that all these results are obtained in the limit.

Mixed strategy equilibria are defined by the condition that firms are indifferent between all prices that are chosen with positive probability or to which a positive density is attached. Let $F_i(.)$ be firm i's price distribution function, with the convention that the mass or density at the price $p_i = p$ itself is not included, that is: $F_i(p) \equiv \Pr[P_i < p]$, where P_i is the random variable from which p is drawn.[41] Let S_i be the support of $F_i(.)$.[42]

Lemma 2: Given some SMIT information process, if both firms have a positive customer base, $F_1(.)$ and $F_2(.)$ have the same support S ($S_1 = S_2 \equiv S$). S is convex, the maximum of S is the monopoly price 1, and the minimum (denoted by $\underline{p}$) lies in the interval $\left(\max\{c_1, c_2\}, 1\right)$.

Lemma 3: Given some SMIT information process, if both firms have a positive customer base, there is no equilibrium where more than one firm attaches positive probability mass to

[40] A firm has a 'positive' customer base if it's previous market share is bounded away from zero.

[41] Under this convention, $1 - F_i(1)$ is the probability mass at the monopoly price.

[42] S_i is the smallest closed set of prices p whose complement has a probability of zero.

any single price. Furthermore, if a mass point is part of a firm's equilibrium strategy, it is located at the monopoly price 1.

Suppose, an equilibrium in mixed strategies exists. Given firm i's price distribution function, firm j's *ex-ante* expected profit (for $p_j = p$) equals:

$$\pi_j = (p - c_j)\left(l(n_j)F_i(p) + h(n_j)\left(1 - F_i(p)\right)\right) \tag{3}$$

In (3), $l(n_j)F_i(p)$ is firm j's demand when it is the high-priced firm times the probability of this event.

By Lemma 1, the right-hand-side in (3) must be independent of $p \quad \forall p \in S$. (3) can be solved for firm i's price distribution function ($i, j \in \{1,2\}$, $j \neq i$):

$$F_i(p) = \frac{h(n_j) - \pi_j / (p - c_j)}{h(n_j) - l(n_j)} \tag{4}$$

By Lemma 2 and 3, the following conditions must be fulfilled:

$$\exists\, \underline{p}:\ F_i(\underline{p}) = 0\ ,\ i \in \{1,2\}\ \text{, and} \tag{5}$$

$$\left(\text{(i) } F_1(1) < 1 \ \wedge\ F_2(1) = 1\right) \ \vee\ \left(\text{(ii) } F_1(1) = 1 \ \wedge\ F_2(1) < 1\right) \ \vee\ \left(\text{(iii) } F_1(1) = F_2(1) = 1\right) \tag{6}$$

Under case (i) in (6), firm 1's price distribution function has a mass point at the monopoly price 1, but not firm 2's. Under case (ii), the situation is reversed. We would normally expect the firm with the smaller customer base to price more aggressively, so the distribution function of the firm with the *larger customer base* should have a mass point. Below, it is shown that this intuition is correct only when firms have identical marginal costs.

To simplify the exposition, suppose from now on that when marginal costs are not identical, firm 1 is always the more *efficient* firm. Without loss of generality, firm 1's marginal cost is normalized to zero ($c_1 \equiv 0$). For an ease of notation, let $c \equiv c_2$, with $c \in [0,1)$. Therefore, c is the difference in marginal costs. Furthermore, let n be the size of firm 1's customer base ($n_1 \equiv n$, $n_2 = 1 - n$). Note, that n fully describes the distribution of past market shares.

Using (3) and (5), we obtain for the lower boundary of the support S:

$$\underline{p} = \frac{\pi_1}{h(n)} \tag{7}$$

Using (3) and (5) once more, we obtain an equation that links π_1 and π_2:

$$\frac{\pi_1}{h(n)} = \frac{\pi_2}{h(1-n)} + c \tag{8}$$

Lemma 4: Given some SMIT information process, there is a critical value for n (denoted by ν) such that $F_2(.)$ has a mass point at $p=1$ if $n<\nu$, and $F_1(.)$ has a mass point at $p=1$ if $n>\nu$. ν solves:

$$(1-c)h(\nu)=h(1-\nu) \tag{9}$$

The solution to (9) is unique, and ν is increasing in c. If $c=0$, then $\nu=1/2$.

Lemma 4 can be restated as follows. If firms have identical marginal costs, the distribution function of the firm with the larger customer base has a mass point at the monopoly price 1. If firms have non-identical marginal costs, the distribution function of the efficient firm (firm 1) has a mass point at $p=1$ if its past market share n is greater than the critical value ν, where ν is increasing in the difference between marginal costs c. If the critical value ν is greater than 1, firm 2's distribution function has a mass point at $p=1$ for any possible value of n.[43] The intuition behind this is, that the inefficient firm finds it hard to defend its customer base, so it tends to exploit the uninformed among its previous customers by choosing the monopoly price with positive probability.

By Lemma 4, if $n\geq\nu$ holds, the condition $F_2(1)=1$ can be used to compute the equilibrium profits. Using (4), one obtains:

$$\pi_1^{n\geq\nu}=l(n) \tag{10}$$

If $n<\nu$, use $F_1(1)=1$ in (4) to obtain:

$$\pi_2^{n<\nu}=(1-c)\cdot l(1-n) \tag{11}$$

Analog expressions for $\pi_2^{n\geq\nu}$ and $\pi_1^{n<\nu}$ follow from (8). The results can be summarized as follows:

$$\pi_1=\begin{cases} l(n) & ,n\geq\nu \\ (1-c)\dfrac{h(n)l(1-n)}{h(1-n)}+c\cdot h(n) & ,n<\nu \end{cases},\quad \pi_2=\begin{cases} \dfrac{h(1-n)l(n)}{h(n)}-c\cdot h(1-n) & ,n\geq\nu \\ (1-c)\cdot l(1-n) & ,n<\nu \end{cases} \tag{12}$$

(4), (9), and (12) give a complete description of the equilibrium for an arbitrary SMIT information process.

Interestingly, by (12), firm 1's expected profit is independent of firm 2's marginal cost c as long as $n\geq\nu$ holds. Therefore, by (4), firm 2's price distribution function is independent of firm 2's own marginal cost. As long as $n\geq\nu$ holds, an increase in c shifts firm 1's price

[43] The case $\nu<0$ never occurs as ν is increasing in c.

distribution function towards lower prices[44], while firm 1's profit is not affected and firm 2's profit decreases since the probability that it loses market shares (compared to the past) is raised and due to the increased production costs.

Lemma 4 stated an important result concerning the identity of the firm that chooses the monopoly price 1 with positive probability. The following Proposition summarizes further predictions about the relative location of the price distribution functions that can be made, depending on the relation between n and ν.

Proposition 1: Given some SMIT information process, if both firms have a positive customer base, the distribution function of the firm with the larger customer base first order stochastically dominates the competitor's distribution function if $c = 0$. If $c > 0$, the distribution function of the inefficient firm (firm 2) first order stochastically dominates firm 1's if $n < \nu$.

Corollary 1: If $c = 0$, the firm with the smaller customer base is likely to gain market shares compared to the past, where 'likely' means with probability greater than 1/2. If $c > 0$, the efficient firm (firm 1) is likely to gain market shares if $n < \nu$.

Note, that since ν increases in c (so $\nu \geq 1/2$), the stochastic dominance result of Proposition 1 holds for at least half of all possible distributions of past market shares. Only if $c > 0$ and $n > \nu$, there is no stochastic dominance. The price distribution functions, then, always have an intersection point. This is illustrated for a numerical example in Figure 1 (using the word-of-mouth information process introduced in Section 2.3).

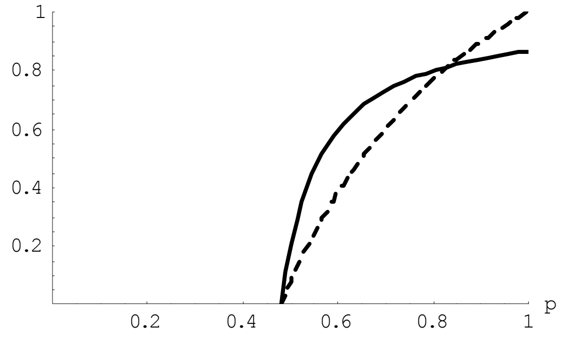

Figure 1: price distribution functions; c = 0.4, λ = 0.5, ρ = 0, n = 0.98; $F_2(p)$: dashed

[44] This can be shown formally using (4), (7), (10), and (12).

Figure 1 shows that the efficient firm (firm 1) chooses the monopoly price with positive probability when $n > \nu$, but it also attaches more weight to prices near $\underline{p}$ than firm 2.

The following Proposition concerns the behavior of the model in the limit as $n \to 1$. This case is used to describe *entry* by the less efficient firm 2.

Proposition 2: Given some SMIT information process, if $c \geq l(1)$ and $n \to 1$ (entry), the incumbent (firm 1) conducts *limit pricing*, where 'limit pricing' means that firm 1 chooses $p = c$ with probability one, and the entrant obtains zero market share.

Proposition 2 implies that entry does not occur in equilibrium if the difference in marginal costs is sufficiently large. If the condition $c \geq l(1)$ is fulfilled, but firm 2 has a positive customer base, the limit pricing result of Proposition 2 does not apply. However, for this case, we can still predict that the market at least *tends* to become more concentrated, because the inefficient firm loses market shares with probability greater than 1/2 for all $n < 1$. This follows from Proposition 1 and from the fact that the condition $c \geq l(1)$ is equivalent to $\nu \geq 1$ (see the Proof or Proposition 2).

2.3. THE PAST SALES – CURRENT PROFIT RELATION UNDER WORD-OF-MOUTH

In the previous section, I derived the equilibrium for the duopoly case for a general information process and identified its main properties. The parameter n that represents the size of firm 1's customer base played a central role in the characterization of the equilibrium. In this section, I analyze whether firms *benefit* from having a large customer base, as economic intuition would suggest. To this end, the information process with word-of-mouth communication is introduced. The results of Section 2.2 are, then, used to analyze the shape of the past sales – current profit relation for this specific information process. It turns out that this relation is not generally monotone. There is a surprising variety of possible shapes.

The information process is based on the following assumptions. Consumers are heterogeneous with respect to their search behavior. Let λ be the fraction of *word-of-mouth consumers* in the population. Each of them is informed about the current offer of her previous supplier, and, in addition, asks one other randomly chosen consumer about his previous choice and finds out about the respective firm's current offer.[45] Let ρ be the fraction of consumers of the *ignorant type*. These consumers simply return to their previous supplier and buy if the price is not greater than 1. The fraction of *searchers* in the population equals $1-\lambda-\rho$. Searchers are aware of all firms' offers and may, e.g., obtain their information from consumer reports (active non-sequential search).[46] Consumers who find out about both firms' offers buy the cheaper product. If prices are identical, they decide for the one they have purchased previously.[47] By assumption, λ and ρ are exogenously fixed, and the customer base of each firm contains the same fraction of word-of-mouth consumers and ignorant types.[48] Under these assumptions, the mass of consumers in a firm's customer base of size n who find out about the other firm's offer equals:

$$(1-\rho)n-\lambda n^2 \tag{13}$$

λn^2 is the mass of word-of-mouth consumers who ask a consumer within the same customer base. If λ and ρ are both close to zero, most consumers are fully informed, so the firm that charges the lower price will serve almost the entire market. If many consumers rely on word-of-mouth, the 'popularity weighting' effect arises since word-of-mouth consumers are more likely to meet a consumer from a large customer base.

Under the above assumptions, the functions $l(.)$ and $h(.)$ introduced in Section 2.2 read:

$$l(n)=\rho n+\lambda n^2 \quad \text{and} \quad h(n)=1-\rho(1-n)-\lambda(1-n)^2 \tag{14}$$

It is easy to check that this is a SMIT information process if $0<\lambda+\rho$ and $\rho<1$.

Using (14), an explicit solution to (9), that defines the critical value ν, is obtained:

$$\nu=\left(\sqrt{4(\lambda+\rho)^2(1-c)+(4\lambda+\rho^2)c^2}-2(\lambda+\rho)+c(2\lambda+\rho)\right)/2c\lambda \tag{15}$$

To apply the results of Propositions 1 and 2, it should be noted that, given (14), the condition $c\geq l(1)$ reads: $c\geq\lambda+\rho$. The value of c for which $\nu=1$, thus, equals $\lambda+\rho$. By Proposition 2, entry into the market is impossible if the entrant's marginal cost exceeds this value.

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[45] Once a consumer has found out about the existence of the rival firm's product, the location of the supplier, and/or the product's characteristics, a price quote can be obtained for free.

[46] Searchers are assumed to pass on information to word-of-mouth consumers only about the product they previously purchased. Results are qualitatively the same if they pass on information about all firms' offers.

[47] Intergenerational aspects can be incorporated by assuming that young consumers who enter this market inherit the information of their parents.

[48] In Section 2.4, an endogenization of λ is introduced for the duopoly case, assuming that consumers are identical and that active search is not prohibitively costly.

In the following, the shape of the past sales – current profit relation is analyzed given the above information process. The results are obtained using particular parameter values. By continuity, each result extends readily to some *range* of parameter values.[49]
First consider the case where marginal costs are identical ($c = 0$). Figure 2 shows firm 1's expected profit as a function of the size of it's customer base (n) for $\lambda = 0.1$ and $\rho = 0$.[50, 51]

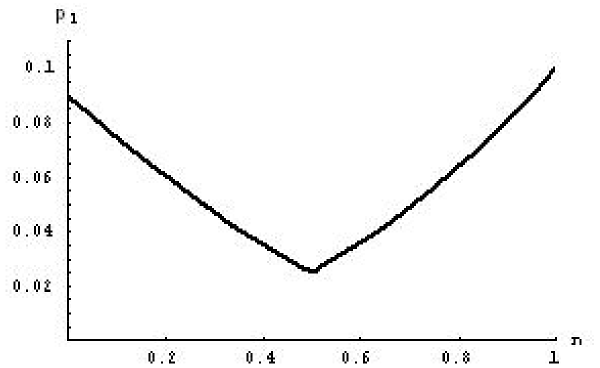

Figure 2: π_1 as a function of n; c = 0, λ = 0.1, ρ = 0

Observation 1: Given the information process specified in (14), there is a range of parameter values for which the past sales – current profit relation is V-shaped, with the minimum at $n = 1/2$.

According to Observation 1, profits are minimized when each firm served half of the market in the past. Firms, thus, prefer to have either a small or a large customer base. Or, to put it another way, a firm that served half of the consumers in the past would be willing to give up its entire customer base and transfer it to the competitor if it could do so without cost. Interestingly, the V-shaped relation is obtained when most consumers are searchers, so we are only a small step away from the standard Bertrand model with full information.[52] The reason for this surprising result is that price competition is more intense for even distributions of past

[49] Proofs are not shown. All results follow directly from (12), (14), and (15).

[50] Note that, if $c = 0$, firms are identical except for the size of their customer base. Therefore, the past sales – current profit relation for firm 2 is simply the mirror picture of Figure 1. It, thus, suffices to plot firm 1's profit over n. If $c > 0$, both firms' profits must be plotted to obtain a complete description of the past sales – current profit relation.

[51] Similar curves as in Figure 2 are obtained for $\lambda = 0$ if ρ is small.

[52] If λ and ρ approach zero, prices converge to zero, but the V-shape is maintained.

market shares. This can be confirmed by comparing the location of the price distribution functions for different values of n.[53]

To understand the intuition why price competition is less intense for skewed distributions of past market shares, note that, if past market shares become more skewed (in a comparative statics sense), the firm with the large customer base (say, firm 1) can attract fewer consumers from firm 2's customer base (by choosing the lower price), while firm 2 can attract more consumers from firm 1. When there are word-of-mouth consumers, the popularity weighting effect must be taken into account. For skewed distributions of past market shares, a higher *fraction* of firm 2's customers find out about firm 1's offer, but the total mass of informed consumers in firm 2's customer base is still decreasing in n (vice versa for firm 1). Therefore, firm 1 always has weaker incentives, and firm 2 has stronger incentives to price aggressively as n increases. It turns out that the first effect dominates, so overall, price competition is less intense for skewed distributions of past market shares.[54]

Figure 3 shows the past sales – current profit relation for $\lambda = 1$ and $\rho = 0$.

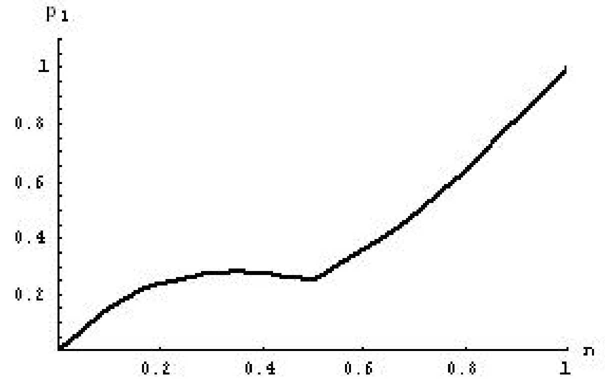

Figure 3: π_1 as a function of n; c = 0, λ = 1, ρ = 0

Observation 2: Given the information process specified in (14), there is a range of parameter values for which the past sales – current profit relation comprises a local maximum.

To understand the intuition behind this result, remember that price competition is less intense for skewed distributions of past market shares. Therefore, starting from an even split of the market, a firm benefits from a reduction in the size of its customer base. However, as its

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[53] They are located further "to the right" (towards higher prices) when n differs from 0.5. It can easily be confirmed that the lowest price ($\underline{p}$) is increasing in n for $n \geq 1/2$.

[54] Note, that prices are strategic complements.

customer base becomes small, the firm is in a disadvantageous position when most consumers rely on word-of-mouth since few consumers find out about the firm's offer. This explains the presence of a local maximum for large values of λ.[55]

Now consider the case with non-identical marginal costs ($c > 0$). As Figure 2 and 3 illustrated, there is usually a kink in the past sales – current profit relation at the critical point ν. Since ν shifts to the right as the difference between marginal costs c increases, there is a variety of shapes of the past sales – current profit relation that can be generated. E.g., the local maximum in Figure 3 can become a *global* maximum if ν is greater than 1.[56] In contrast to Observation 1, firms, then, prefer even splits of the market.

The following graph was obtained for $c = 0.1$, $\lambda = 0.1$, $\rho = 0$, and shows an even more surprising result.

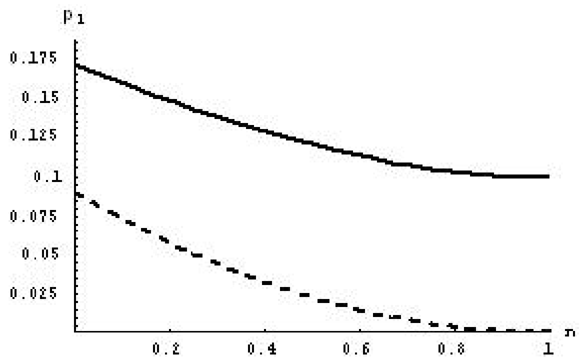

Figure 4: π_1 and π_2 as functions of n; c = 0.1, λ = 0.1, ρ = 0; π_2: dashed

Observation 3: Given the information process specified in (14), there is a range of parameter values for which the *efficient* firm's past sales – current profit relation is monotonically *decreasing*.

Observation 3 completely reverses the basic intuition that firms should benefit from having a large customer base. The efficient firm (firm 1) prefers to have no customer base at all. The expected profit of the inefficient firm, however, increases in the size of its customer base ($1-n$). As before, the result stems from the altered intensity of price competition as the past distribution of market shares is changed.

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[55] In Figure 3, firm 1's profit approaches zero for $n \to 0$. This holds for $\lambda = 1$, because when everybody relies on word-of-mouth, nobody will find out about the offer of a firm with zero sales in the past. Entry is, then, impossible unless the entrant finds other ways to spread the information (like advertising).

[56] This is e.g. the case for $\lambda = 0.5$, $\rho = 0$, $c = 0.5$.

The above observations show that the most counter-intuitive results are obtained for small values of λ and ρ, that is, when most consumers are fully informed. It is, however, reassuring that also parameter values exist for which our basic intuition is confirmed, that is, where each firm benefits from an increase in the size of its own customer base.[57]

2.4. ENDOGENOUS WORD-OF-MOUTH

In Section 2.3, I assumed that the characteristics of the market, in particular the size-distribution of firms' customer bases, have no impact on consumers' search decisions. This is plausible if consumers are poorly informed about the relevant characteristics of the market. The fractions of word-of-mouth consumers and consumers of the ignorant type in the population were, thus, treated as exogenous. In the following, I will instead assume that consumers are informed about the characteristics of the market and use their information to decide rationally whether to search actively or to communicate via word-of-mouth. The question that shall be tackled is under what conditions rational consumers decide to rely on word-of-mouth.

Suppose, word-of-mouth communication is for free, while active search is costly. Word-of-mouth may be interpreted as a free by-product of social interaction. Therefore, there will be no consumers of the ignorant type in equilibrium.[58] The analysis in this section is restricted to the duopoly case with identical marginal costs ($c = 0$, so $\nu = 1/2$ by Lemma 4). Consumers are assumed to be identical except for the identity of their previous supplier. They are aware of the number of suppliers in the market and their cost functions, as well as about the total mass of consumers and their uniform reservation price. As before, every consumer can obtain a price quote from her previous supplier for free, and find out about the other firm's offer by active search or via word-of-mouth.

Let the search cost be $z > 0$, and denote by L the expected loss of utility due to incomplete information when the consumer does *not* search actively. Rational consumers, thus, search actively if $L \geq z$. Without loss of generality, let firm 1 be the firm with the larger customer

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[57] This is e.g. the case for $c = 0$, $\lambda = 0$, $\rho = 0.7$ (as an example with many ignorant types), or for $c = 0.7$, $\lambda = 0.5$, $\rho = 0$ (as an example with a large difference in marginal costs).

[58] This is only to simplify the exposition. To obtain an equilibrium with consumers of the ignorant type and word-of-mouth consumers, one must assume that word-of-mouth is costly, too (at least for some consumers), but not as costly as active search. To obtain an equilibrium only with consumers of the ignorant type and searchers, use $l(n) = \rho n$ instead of $l(n) = \lambda n^2$ in the formula for L derived in the Appendix. Equation (16) (below) must also be modified. All other computations remain valid.

base, and let $n \equiv n_1$ be the size of its customer base (so $n \geq 1/2$). Since L depends on n, consumers must have some prior beliefs about the distribution of past market shares. For simplicity, suppose that consumers know the value of n. To obtain an equilibrium where the same fraction of consumers in both firms' customer bases decide to search actively, I assume that consumers do not know whether their previous supplier was the one with the larger or the smaller customer base. L, thus, equals:

$$L = \sum_{i=1}^{2} n_i^2 \cdot \Pr\left[P_i > P_j\right] \cdot E\left[P_i - P_j \mid p_i > p_j\right] \tag{16}$$

n_i is the probability that the consumer belongs to firm i's customer base. The probability that he asks another consumer from firm i's customer base and, thus, remains ignorant about firm j's ($j \neq i$) offer is also equal to n_i. This yields n_i^2.[59] P_i is the random variable from which firm i's price is drawn, and $\Pr\left[P_i > P_j\right]$ is the probability that firm i charges the higher price in the market. (Otherwise, there is no disadvantage in being ignorant about firm j's offer.)
The computation of L is shown in the Appendix. The expression that is derived can be evaluated numerically. Figure 5 shows the dependency of L from n and λ in a 3-D-plot.

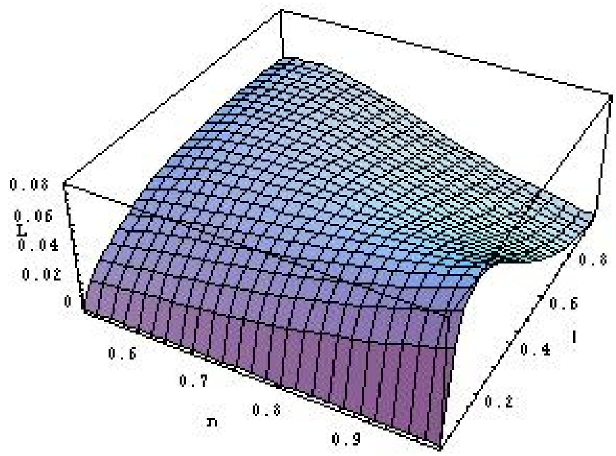

Figure 5: L as a function of n and λ

Figure 5 illustrates that L increases in n and λ unless n and λ are sufficiently large. This can be explained as follows. When past market shares become more skewed or when more consumers rely on word-of-mouth, price competition gets softer. Starting from an even

[59] When word-of-mouth communication is assumed to be prohibitively costly, and the fraction of ignorant types (ρ) rather than λ is to be endogenized, n_i^2 is to be replaced by n_i in (16).

distribution of past market shares (n near 1/2) and a low fraction word-of-mouth consumers, this leads to an increase in the expected price differential. However, when n and λ are sufficiently large, both prices approach the monopoly price level, so the expected price differential and, thus, L decreases when n or λ is increased further.
The equilibrium values for λ are defined by the condition:

$$L(n,\lambda) = z \tag{17}$$

They correspond to the intersections of the L - curve (for a fixed value of n) and a horizontal line at the level z. If z is sufficiently large, there are no intersection points. Active search is, then, too costly so all consumers rely on word-of-mouth ($\lambda = 1$). If there are two intersections, a simple argument reveals that the larger equilibrium value for λ is *unstable*. At this point, the L - curve is decreasing in λ. If a searcher deviates and becomes a word-of-mouth consumer, λ increases, so the expected loss due to incomplete information for all word-of-mouth consumers decreases. Therefore, more searchers will deviate and become word-of-mouth consumers, and so on. There is, thus, another stable equilibrium at $\lambda = 1$, that is referred to as the "*no-search-equilibrium*". To see this, note that, when everybody believes that nobody will search actively, expected prices are close to the monopoly price, so the expected price differential is small. Therefore, nobody has an incentive to search actively unless z is sufficiently small. Figure 6 shows the equilibrium values for λ, for $n = 0.7$.

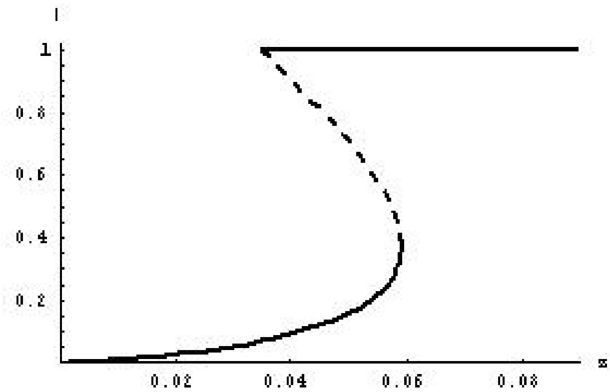

Figure 6: endogenous λ as a function of z, n = 0.7; unstable equilibrium: dashed

To see how profits are affected by changes in the search cost z, note that the expected profit of the firm with the larger customer base (firm 1) is increasing in λ for all values of $n \geq 1/2$ (this follows immediately from (12) and (14)). It can further be shown that firm 2's expected profit is increasing in λ unless n and λ are sufficiently large. Therefore, if we are on the increasing branch in Figure 6 where an increase in z leads to an increase in λ and, thus, to less active search, in most cases, both firms will benefit from the increase in z. Only if n and

λ are sufficiently large, an increase in z that leads to an increase in λ may reduce firm 2's profit. This is because a firm with a very small customer base is in an unfavorable position in a market where most consumers rely on word-of-mouth, so it prefers more active search. If we are on the horizontal line in Figure 6, that represents the no-search equilibrium, an increase in z has no effect upon the amount of search and, thus, upon profits. However, a small *reduction* in z can induce a large amount of active search if the reduction goes beyond the point where the no-search-equilibrium exists, which adversely affects profits.

Although an endogenous determination of λ clearly affects the shape of the past sales – current profit relation that was the focus of Section 2.3, the basic non-monotonicity result is preserved. To see this, suppose e.g. that the search cost z is sufficiently small, so that the no-search equilibrium does not exist for most values of n (see Figures 5 and 6). Therefore, a large fraction of consumers will search actively unless n is close to 1, so λ is small for most values of n. The past sales – current profit relation will, thus, have the V-shape that is typical for the case with almost full information.

2.5. THE OLIGOPOLY CASE

This section contains an extension of the model introduced in Sections 2.2 and 2.3 to oligopolies with more than two firms. The following questions will be tackled: 1. Do prices and profits converge to zero as the number of competitors becomes infinite? And 2.: Do the non-monotonicities in the past sales – current profit relation persist when the number of firms is raised? The analysis is restricted to situations where all firms have the same marginal cost (normalized to zero), and the information process introduced in Section 2.3 is used.

Let $M \geq 2$ be the number of competitors in the market. Under the assumptions of Section 2.3, the mass of consumers in firm i's customer base who find out about firm j's offer equals ($i, j \in \{1, 2, ..., M\}$, $j \neq i$):

$$(1-\lambda-\rho)n_i + \lambda n_i n_j \tag{18}$$

Note, that while in the duopoly case, firm 1's past market share was sufficient to describe the initial split of the market, $M-1$ parameters are needed in the oligopoly case. To obtain tractable results, only two special cases are analyzed. In the *symmetric* oligopoly case, all firms had the same market share in the past.[60] This case is used to analyze prices and profits as the number of firms approaches infinity. In the *asymmetric* oligopoly case, all firms *but one*

[60] This is essentially the case that has been studied extensively in the search literature. The only difference is that the present model incorporates word-of-mouth communication.

had the same market share in the past. This is used to characterize the shape of the past sales – current profit relation in oligopoly.

2.5.1. The symmetric oligopoly case

Let $n \equiv n_1 = ... = n_M$ be the mass of consumers per firm. When the number of firms (M) is varied, it is sometimes desirable to keep the mass of consumers per firm constant. The total mass of consumers (the market size), in the following denoted by N, must then be adjusted:

$$N = n \cdot M \tag{19}$$

A symmetric equilibrium in mixed strategies is derived.[61] Let $F(.)$ be the equilibrium price distribution function of all firms, and let $G(p) \equiv 1 - F(p)$. Consider firm i playing against $M-1$ competitors that randomize according to $F(.)$. Firm i's expected profit is:

$$\pi = pn\left(\lambda / M + \rho + 2\lambda(M-1) \cdot G(p)/M + (1-\lambda-\rho)M \cdot G(p)^{M-1}\right) \tag{20}$$

, where $n\lambda / M$ is the mass of word-of-mouth consumers in firm i's customer base who ask a consumer within the same customer base, and ρn is the mass of consumers of the ignorant type in firm i's customer base. $(n\lambda / M)G(p)$ is the mass of word-of-mouth consumers in firm i's customer base who ask a consumer from a competitor's customer base times the probability that this firm charges a higher price than firm i.[62] Since firm i has $M-1$ competitors, this must be multiplied by $M-1$. It is further multiplied by 2 to account for the competitors' word-of-mouth consumers who switch to firm i, since there are also $n\lambda / M$ consumers in each of the competitors' customer bases who ask a consumer from firm i's customer base. The expression $n(1-\lambda-\rho)M \cdot G(p)^{M-1}$ is the total mass of searchers in the population times the probability that firm i charges the lowest price in the market.

In a symmetric equilibrium, it must hold that $F(1) = 1$, so $G(1) = 0$. Therefore, one obtains for the equilibrium profit per firm (using (20)):

$$\pi = n\left(\lambda / M + \rho\right) \tag{21}$$

Using (20) and (21), the condition $G(\underline{p}) = 1$ yields for the minimum of the support of $F(.)$:

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[61] Baye, Kovenock, and de Vries (1992) show that in Varian's (1980) model, asymmetric equilibria can exist for $M > 2$, but the only equilibrium that is consistent with an intuitive equilibrium refinement is the symmetric one. For the model presented in this chapter, such asymmetric equilibria can only exist if $\lambda = 0$ (not shown).

[62] Remember, that each word-of-mouth consumer finds out about at most two offers (one of them being the offer of her previous supplier).

$$\underline{p} = \frac{\lambda + \rho M}{\lambda + \rho M + 2\lambda(M-1) + (1-\lambda-\rho)M^2} \quad (22)$$

Proposition 3: If $\rho = 0$, aggregate industry profit is independent of the number of firms if the mass of consumers per firm (n) is held fixed, while prices converge to the competitive price as $M \to \infty$.

While prices and individual profits converge to zero for $M \to \infty$ when there are only word-of-mouth consumers and searchers, by Proposition 3, industry profit remains constant when the market size is adjusted such that the mass of consumers per firm remains constant. In terms of industry profit, when M increases, the intensified price competition and increased market size are two forces that cancel each other out. Note, that, when there are infinitely many firms, the probability that a consumer asks another consumer from the same customer base is zero, so all consumers are aware of at least two offers. This drives prices down to zero. Different results are obtained when there are consumers of the ignorant type and searchers:

Proposition 4: If $\lambda = 0$, aggregate industry profit is independent of the number of firms if the market size (N) is constant, while prices converge to the monopoly price as $M \to \infty$.

It is surprising to note that, while prices converge to the monopoly price as $M \to \infty$ for $\lambda = 0$ (Proposition 4), the lowest price ($\underline{p}$) does not. Using (22), it can easily be confirmed that $\underline{p}$ converges to *zero* as $M \to \infty$. To understand the intuition behind this, it is useful to plot the density function $F'(.)$ for a finite number of firms. When λ equals zero, the distribution function $F(p)$ can easily be solved for (see the Proof of Proposition 4):

$$F(p) = 1 - \left(\frac{\rho / p - \rho}{(1-\rho)M} \right)^{1/(M-1)} \quad (23)$$

The density function $F'(.)$ is simply the first derivative of $F(.)$. Figure 7 shows $F'(p)$ for $M = 8$ and $\rho = 0.1$.

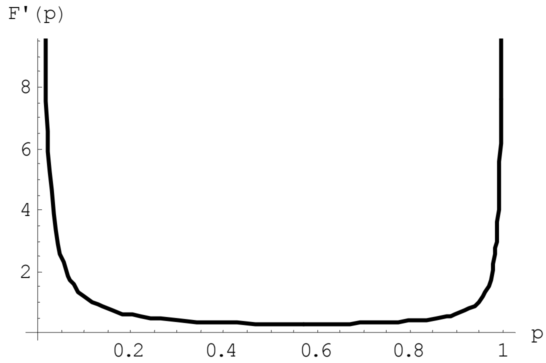

Figure 7: density function; M = 8, λ = 0, ρ = 0.1

As Figure 7 illustrates, firms choose low prices (close to the competitive price) and high prices (close to the monopoly price) with high probability, while the density at intermediate prices is low. This can be explained as follows. Firms have an incentive to exploit the ignorant types among their previous customers, which makes the choice of high prices attractive. On the other hand, they can try to undercut the competitors' prices to attract the searchers. To achieve this with "reasonable" probability, a firm must choose a sufficiently low price. This explains why intermediate prices are chosen with lower probability than extreme prices. When the number of firms gets large, the chance to become the firm with the lowest price is small, so it becomes relatively more profitable to exploit the ignorant types. This explains why the density converges to zero for all prices below 1 as $M \to \infty$. That the industry profit does not increase in M despite the fact that prices converge to the monopoly level is due to the fact that the *average price charged* is constant. Individual firms choose higher prices, but since there are more firms, the expected lowest price falls in M.[63]

The results of Proposition 3 and 4 can be compared to a result from the search literature, according to which markets may become less competitive as the number of suppliers increases (see Rosenthal (1980), Stiglitz (1987), Stahl (1989), Janssen and Moraga-Gonzalez (2004)). Proposition 4 reproduces this result for the case with searchers and consumers of the ignorant type, but it is not obtained for the case with searchers and word-of-mouth consumers.

[63] For $\lambda = 0$, industry profit equals $N\rho$ (see (21)), which is simply the industry profit when all searchers pay a price of zero, and consumers of the ignorant type pay the monopoly price.

2.5.2. The asymmetric oligopoly case

Let $n \equiv n_1$ be the size of firm 1's customer base, and let $\bar{n} \equiv n_2 = ... = n_M$ be the size of each of the other firms' customer bases.[64] Let the market size be constant ($N \equiv 1$). Therefore, we have:

$$\bar{n} = \frac{1-n}{M-1} \tag{24}$$

An asymmetric equilibrium in mixed strategies is derived. Let $F_1(.)$ be firm 1's, and let $F(.)$ be all other firms' equilibrium price distribution function. Let $G_1(p) \equiv 1 - F_1(p)$, and $G(p) \equiv 1 - F(p)$. When all other firms randomize according to $F(.)$, firm 1's expected profit equals: [65]

$$\pi_1 = p\left(\lambda n^2 + \rho n + 2\lambda n\bar{n}(M-1)G(p) + (1-\lambda-\rho)G(p)^{M-1}\right) \tag{25}$$

Now consider one of the other firms (firm i, $i \in \{2,3,...,M\}$). When all competitors (including firm 1) randomize according to their equilibrium distribution function, firm i's expected profit is:

$$\pi = p\left(\lambda\bar{n}^2 + \rho\bar{n} + 2\lambda\bar{n}^2(M-2)G(p) + 2\lambda n\bar{n} \cdot G_1(p) + (1-\lambda-\rho)G(p)^{M-2}G_1(p)\right) \tag{26}$$

Two cases must be distinguished: 1. $n \geq 1/M$, and 2. $n < 1/M$.

First consider the case $n \geq 1/M$. If n is strictly greater than $1/M$, firm 1 has the largest customer base ($n > \bar{n}$), and $F_1(.)$ first order stochastically dominates $F(.)$. By an analog of Lemma 3, $F(.)$ can not have a mass point, while $F_1(.)$ has a mass point at $p = 1$ when $n > 1/M$.[66] The following condition is, thus, used to characterize the equilibrium: $F(1) = 1 \Leftrightarrow G(1) = 0$. Using (25), this yields:

$$\pi_1 = n\left(\lambda n + \rho\right) \tag{27}$$

Note, that by (27), π_1 is *monotonically increasing* in n.[67] The following Proposition follows immediately from the fact that (27) holds for all $n \geq 1/M$:

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[64] n is, thus, sufficient to describe the size-distribution of the firms' customer bases

[65] see the explanation of equation (20)

[66] Formal proofs are not shown here.

[67] That firm 1's profit is increasing *quadratically* in the size of its customer base for $n \geq 1/M$ and $\lambda > 0$ can be explained by two effects that occur simultaneously: 1. An increase in n yields a higher profit at fixed prices, and 2. price competition becomes softer when n increases since firm 1's competitors become "weaker" (fewer customers find out about their offer via word-of-mouth).

Proposition 5: Under the word-of-mouth information process, all non-monotonicities disappear from the past sales – current profit relation as $M \to \infty$.

Proposition 5 gives an answer to the question whether the non-monotonicities in the past sales – current profit relation that were observed in the duopoly case can persist for the case where the number of firms becomes infinite. They can not. The intuition behind this result is as follows. Non-monotonicities arise if a reduction in the size of a firm's customer base induces softer price competition and this more than compensates the firm for the loss of monopoly power over repeat purchasers. When there are many firms, the reduction has a smaller effect upon the competitors' pricing behavior since they are also engaged in a struggle for each other's customers. As a result, the non-monotonicities disappear.

Proposition 5 leaves open the question whether the non-monotonicities already disappear when there is only a finite number of firms (greater than two). It is clear that the non-monotonicities must at least become less significant when M is increased, as the interval where the past sales – current profit relation *can* be decreasing (the interval from 0 to $1/M$) becomes smaller. To obtain general results, the case $n < 1/M$ must be analyzed.

It can be shown that for $n < 1/M$, neither $F(.)$ nor $F_1(.)$ can have a mass point. Furthermore, $F(.)$ first order stochastically dominates $F_1(.)$. An analog of Lemma 2, however, does not obtain for the case $n < 1/M$. More specifically, for $n < 1/M$, the support of $F(.)$ and the support of $F_1(.)$ do *not* have the same maximum. While the former is still the monopoly price 1, the maximum of the support of $F_1(.)$ (denoted by σ) lies within the interval $(\underline{p}, 1)$.

The easiest way to proceed is to compute a candidate equilibrium, and to verify that it fulfills $\underline{p} < \sigma < 1$.[68] The candidate equilibrium is computed as follows. Denote the interval from $\underline{p}$ to σ interval A, and the one from σ to 1 interval B. Let

$$F(p) \equiv \begin{cases} F_A(p) & \text{for } \underline{p} \le p \le \sigma \\ F_B(p) & \text{for } \sigma < p \le 1 \end{cases} \tag{28}$$

be the distribution function of firm 2, 3, …, and M. $G_A(.)$ and $G_B(.)$ are defined accordingly. Figure 8 shows $F(.)$ and $F_1(.)$ for $M = 3$, $\lambda = 0.5$, $\rho = 0$, $n = 0.1$.[69, 70]

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[68] It must also be verified that firm 1 does not benefit from deviating to a price between σ and 1. It can be shown that this is always fulfilled.

[69] To construct Figure 10, the equilibrium must, of course, first be computed; see below.

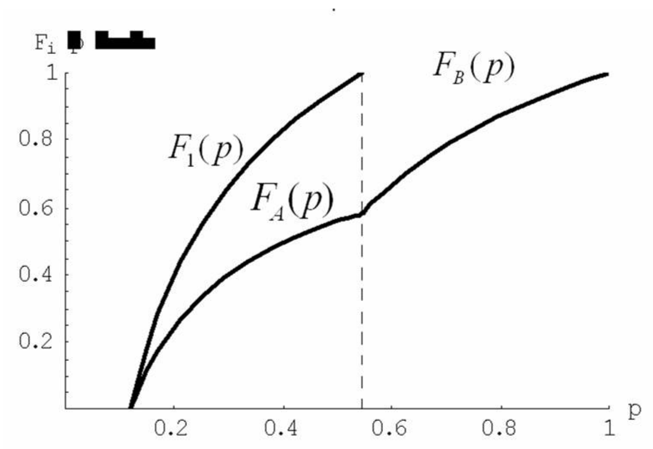

Figure 8: distribution functions, M = 3, λ = 0.5, ρ = 0, n = 0.1

For interval A, equations (25) and (26) are valid if $G(.)$ is replaced by $G_A(.)$. For interval B, (26) is valid for $G_1(.) = 0$. This yields:

$$\pi = p\left(\lambda\overline{n}^2 + \rho\overline{n} + 2\lambda\overline{n}^2(M-2)G_B(p)\right) \tag{29}$$

Using the equilibrium condition $G_B(1) = 0$, (29) yields:

$$\pi = \overline{n}\left(\lambda\overline{n} + \rho\right) \tag{30}$$

Note, that (30) is identical to (27) when $\overline{n}$ is replaced by n.
Using (30) and the equilibrium condition $G_A(\underline{p}) = G_1(\underline{p}) = 1$, (25) and (26) yield:

$$\pi_1 = \overline{n}(\lambda\overline{n} + \rho)\frac{(2\lambda + \rho)n - \lambda n^2 + 1 - \lambda - \rho}{\rho\overline{n} + \lambda\overline{n}^2(2M-3) + 2\lambda n\overline{n} + 1 - \lambda - \rho} \tag{31}$$

All that remains to be done is to compute σ, and to verify that $\underline{p} < \sigma < 1$ holds. Using the condition $G_1(\sigma) = 0$, for $p = \sigma$, (25), (26), and (30) yield two equations from which $G_A(\sigma)$ can be eliminated to yield a single equation that implicitly defines σ. A general solution to this equation can be obtained for the special case without searchers ($\lambda + \rho = 1$) (not shown). For $\lambda + \rho < 1$, a numerical analysis confirmed that $\underline{p} < \sigma < 1$ always holds for $n < 1/M$.

Figure 9 shows firm 1's profit as a function of the size of its customer base (n) as defined by (31) for $n < 1/M$, and (27) for $n \geq 1/M$, for $M = 3$, $\lambda = 1$, and $\rho = 0$.

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[70] Figure 10 illustrates that, when $n < 1/M$, firm 1 tends to choose lower prices than the competitors. When n gets smaller, firm 1's distribution function becomes steeper. It can be shown that, for $n \to 0$ (entry into a market with $M-1$ incumbents), firm 1 chooses $p = \underline{p}$ with probability one if $\lambda = 1$.

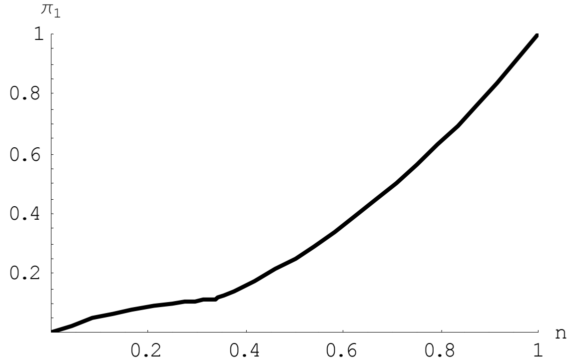

Figure 9: $\pi_1$as a function of n; M = 3, λ = 1, ρ = 0

The comparison of Figure 9 and Figure 3, that were constructed for the same parameters (except M), shows that the local maximum that was obtained in the duopoly case disappears when the number of firms is raised to three. However, for other parameter values[71], non-monotonicities are still obtained. It must, thus, be concluded that non-monotonicities can persist when the number of firms is raised but remains finite.

2.6. DISCUSSION

The discussion addresses two modeling choices that deserve special attention.

The first concerns the use of mixed pricing strategies. The equilibrium derived in this chapter has the peculiar property that *ex-post*, given the price of the competitor (in duopoly), a firm's randomly chosen price is non-optimal *with probability one*. It would always be more profitable for the firm to undercut the competitor's price marginally if the competitor's price were *ex ante* known.[72] However, the fact that this does not occur in equilibrium is the essence of the mixed strategy equilibrium. Since a firm with a predictable pricing behavior can be taken advantage of, firms have an incentive to be unpredictable. Mixed strategy equilibria are, thus, a typical feature of search models with homogeneous products.[73] My decision not to include product differentiation, switching costs, or other features that might lead to pure strategy equilibria can be justified by a principle of parsimony. The goal was to build a model that yields an important improvement over the standard Bertrand model (in particular the inclusion of incomplete information and repeat purchasing), without deviating in more ways than necessary from the standard model.

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[71] e.g. $M = 4$, $\lambda = 0.1$, $\rho = 0$

[72] unless the competitor chooses $p = \underline{p}$, but this event occurs with zero probability in equilibrium

[73] See Varian (1980), and the discussion in Rosenthal (1980), page 1578.

The second point relates to the fact that I explicitly included the history of the market in the model, but not the future. It would, indeed, be desirable to build a full-fledged dynamic model of price competition under word-of-mouth communication. However, static models are widely used in economic theory, and a separate analysis of the static case seems justified in this case. In a dynamic setting, the effects that were found in the chapter would still be present, but additional effects stemming from the incentives to invest or to disinvest in future market shares would be added. However, these additional effects will not overturn all the results from the static analysis. To see this, consider e.g. the case with almost complete information ($\lambda + \rho$ near zero). In this case, market shares are very "volatile" in the sense that the low-priced firm always serves almost the entire market demand, irrespective of the size of its customer base. Therefore, the incentives to *invest* in future market shares are almost negligible, and the V-shaped relation between past sales and current profits is preserved.[74]

Furthermore, the static framework is a good approximation of a more general dynamic model, introduced in the following. Suppose, the products sold in our market have limited durability, and denote the constant failure rate per period by α. The idea is that, even if a firm serves a high fraction of current market demand, the competitor's customer base may still be large if the competitor served most of the market in previous periods, since not every consumer is making a new purchase each period. Word-of-mouth consumers are, thus, likely to ask consumers who's previous purchase is several periods ago, and the size-distribution of firms' total customer bases (including all consumers who purchased from the respective firm when they last made a purchase) is less volatile than in the case where each consumer makes a purchase each period.

Let Ω be the total population size. If all consumers have purchased a product in this market before, the mass of consumers who purchase a product *per period* is $\alpha \cdot \Omega$. In the context of the model introduced in Section 2.2, we, thus, have:

$$\alpha \cdot \Omega = 1 \tag{32}$$

Let Ω_i be the total mass of consumers who purchased firm i's product when they last made a purchase. In the context of the model of Section 2.2, we have:

$$n_i = \alpha \cdot \Omega_i \tag{33}$$

[74] Of course, this depends on the equilibrium concept chosen, since the set of equilibria in a dynamic pricing game is potentially overwhelming. To rule out collusive equilibria, the Markov perfection equilibrium refinement may be imposed.

Let Ω_i' be the total mass of consumers who previously purchased firm i's product, evaluated next period. If D_i is firm i's current demand, we have:

$$\Omega_i' = (1-\alpha)\Omega_i + D_i \tag{34}$$

Using (33) (evaluated next period), (34) can be written as:

$$n_i' = (1-\alpha)n_i + \alpha D_i \tag{35}$$

Let n_i be the state variable of the dynamic game. If the failure rate α converges to zero, and the total population size is adjusted so that (32) remains valid, then, by (35), the state next period is identical to the current state, irrespective of whether firm i is the high- or the low-priced firm in the current period.[75] If future profits are discounted, and collusive strategies are ruled out, e.g. by imposing the Markov perfection equilibrium refinement, then the static model is obtained in the limit as the failure rate α approaches zero. The static model, then, describes competition in a single period of the dynamic game. Intuitively, if the durability of the products is large, then firms find themselves in the same situation (same size-distribution of customer bases) for many periods to come, irrespective of whether they charge the higher or the lower price in the market. Due to discounting, profits accumulate mostly over a limited number of periods, while the size-distribution of customer bases changes only in the distant future. The incentives to invest or disinvest in future market shares, thus, vanish.

2.7. CONCLUDING REMARKS

In markets characterized by repeat purchasing, different types of market imperfection, such as incomplete consumer information or switching costs, can lead to a direct dependency of current profits from past sales. Economic intuition suggests that, if such a link exists, firms are better off with a large customer base. This is generally true if the size-distribution of the firms' customer bases has no effect upon the intensity of price competition. However, if the intensity of price competition is affected, this intuition may be misleading. Firms may have an incentive to *disinvest* in the size of their customer base if this induces softer price competition in the future. The present chapter showed that, in a duopoly with incomplete information, non-monotonic relations between the size of a firm's customer base and profit are obtained for a large set of parameter values. Interestingly, the non-monotonicities are particularly pronounced when most consumers are fully informed, so we are only a small step away from the standard Bertrand model with full information. When the number of firms is raised, the non-monotonicities become less significant, and they disappear when the number of firms

[75] Note, that $D_i \in [0,1]$.

approaches infinity. Furthermore, it was shown that rational consumers rely on word-of-mouth as a free by-product of social interaction if the expected loss due to incomplete information is not greater than the search cost.

The following chapter will extend the model to a full-fledged dynamic game. The results on the shape of the past sales – current profit relation obtained in the static framework promise interesting implications for firms' investment incentives in future market shares, and for the endogenous market share dynamics generated by firms' random pricing strategies.

2.8. Appendix

Proof of Lemma 1:

Note first that since the information process is imperfect and each firm had a strictly positive market share in the past, all prices above marginal cost (including the monopoly price 1) yield a strictly positive profit. Suppose both firms charge an identical price $p \in (\max\{c_1, c_2\}, 1]$. This can not be an equilibrium because each firm would benefit from marginally undercutting p, as this leads to a discontinuous rise in demand since the information process is non-trivial. If both firms choose $p = \max\{c_1, c_2\}$, the profit of the inefficient firm (or both firms when $c_1 = c_2$) is zero. This can not be an equilibrium since higher prices yield strictly positive profit. If $p_1 \neq p_2$ and $p_1, p_2 \in [\max\{c_1, c_2\}, 1)$, the high-priced firm would benefit from deviating to the monopoly price. It can not be an equilibrium for one firm to choose the monopoly price, and for the other to marginally undercut this price, as the high-priced firm would benefit from matching the lower price. It can not be an equilibrium for one firm to choose the monopoly price, and for the other to undercut this price more than marginally, as the low-priced firm would benefit from deviating to a higher price.

Proof of Lemma 2:

Suppose $S_i \neq S_j$ ($j \neq i$). Therefore, for $i = 1$ or for $i = 2$, there is a price $\tilde{p} < 1$, with $\tilde{p} \in S_i$ but $\tilde{p} \notin S_j$. This can not be an equilibrium because, since $\overline{S_j}$ is open, there exists a price $p > \tilde{p}$ with $p \notin S_j$ that yields the same expected demand to firm i as $\tilde{p}$ but a higher profit. This holds if p is above the maximum of S_j (if the maximum is below 1), below the minimum of S_j, or within some intermediate range that is not part of S_j when S_j is not convex. Therefore, $S_1 = S_2 \equiv S$. The maximum of S must be the monopoly price 1, because otherwise, each firm would benefit from deviating to the monopoly price. This yields the same demand as the maximum of S but a higher profit. $\underline{p}$ must be greater than $\max\{c_1, c_2\}$ because $p = \underline{p}$ yields zero profit to the inefficient firm. $\underline{p}$ is smaller than 1 since (by Lemma 1), there is no pure strategy equilibrium. Furthermore, S is convex. Suppose to the contrary that there is an intermediate range that is not part of S. Firm i's expected demand would be constant over this range, but expected profit would be increasing. Therefore, expected profit would be higher in the upper interval of S. This can not be an equilibrium. □

Proof of Lemma 3:

Suppose both firms attach positive probability mass to some identical price level p in $S = [\underline{p}, 1]$. Each firm would, then, benefit from shifting its mass point to a price level marginally below p because this leads to a discontinuous rise in expected demand. Strategies containing a single mass point at some price p in the interval $(\underline{p}, 1)$ can not be an equilibrium either since the competitor's expected demand (and, thus, profit) would fall discontinuously at p. There can be no equilibrium where the distribution function of one firm contains a mass point at $p = \underline{p}$ as the competitor's expected profit would be larger at prices marginally below $\underline{p}$ than at prices above $\underline{p}$. This can not be an equilibrium since prices below $\underline{p}$ are not part of the support.

Proof of Lemma 4:

Case (i) in (6) is obtained if $F_2(1) > F_1(1)$. Using (4), this condition can be written as:

$$(1-c)\big(h(n) - h(1-n)\big) - (1-c)\pi_1 + \pi_2 > 0 \tag{36}$$

Use (8) to replace π_2 by $h(1-n)\big(\pi_1 / h(n) - c\big)$. Then use (7) to replace π_1 by $\underline{p}h(n)$. This yields after rearranging:

$$(1-\underline{p})\big((1-c)h(n) - h(1-n)\big) > 0 \tag{37}$$

Since $\underline{p} < 1$ under the conditions of Lemma 4, this is fulfilled if and only if

$$(1-c)h(n) > h(1-n) \tag{38}$$

Since $c < 1$ and $h(.)$ is strictly increasing, the left-hand-side is strictly increasing, and the right-hand-side is strictly decreasing in n. It follows immediately that there is at most one single value for n (ν) in the interval $[0,1]$ that fulfills $(1-c)h(n) = h(1-n)$ (this yields (9)). If $n > \nu$, condition (38) is clearly fulfilled and case (i) in (6) is obtained. If $n < \nu$, the condition is violated and case (ii) in (6) is obtained (to see this, simply replace all "$>$" signs in the above proof by a "$<$" sign). For $c=0$, (9) simplifies to $h(v)=h(1-v)$. Since $h(.)$ is strictly increasing, this requires that $v=1-v \Leftrightarrow v=1/2$. To show that v is increasing in c, differentiate (9) w.r.t. c and solve for dv/dc. If the information process is strictly monotone and non-trivial, dv/dc is greater than zero for all $v\in[1/2,1]$. □

Proof of Proposition 1:

The second claim (for $c>0$) shall be proved first. First order stochastic dominance of firm 2's over firm 1's price distribution function requires that $F_1(p)\geq F_2(p)$ holds for all $p\in[\underline{p},1]$, with strict inequality for some p for strict dominance. It must be shown that this is fulfilled when $n<v$. To this end, similar steps are used as in the proof of Lemma 4. Using (4) , the above condition can (for strict dominance) be written as:

$$p(p-c)\big(h(1-n)-h(n)\big)+(p-c)\pi_1-p\pi_2>0 \tag{39}$$

Use (8) to replace π_2 by $h(1-n)\big(\pi_1/h(n)-c\big)$, and (7) to replace π_1 by $\underline{p}h(n)$. This yields after rearranging:

$$(p-\underline{p})\big(ph(1-n)-(p-c)h(n)\big)>0 \tag{40}$$

For $p>\underline{p}$, this is equivalent to:

$$ph(1-n)-(p-c)h(n)>0 \tag{41}$$

$ph(1-n)-(p-c)h(n)$ is decreasing in p if the first derivative is smaller or equal to zero: $h(1-n)-h(n)\leq 0 \Leftrightarrow h(n)\geq h(1-n)$. Since $h(.)$ is strictly increasing, this is equivalent to $n\geq 1/2$. Therefore, if condition (41) is fulfilled for $p=1$, it is also fulfilled for all other prices in the interval $(\underline{p},r]$. Given that $n\geq 1/2$, the condition can, thus, be rewritten as:

$$h(1-n)-(1-c)h(n)>0 \tag{42}$$

By Lemma 4, this is equivalent to $n<v$.

If $n<1/2$, then $h(1-n)>h(n)$. Since $p>c$, condition (41) is, thus, fulfilled.

To show the first claim (for $c=0$), note that the above proof already covers the case where $n<1/2$. For $n\geq 1/2$, replace all "$>$" signs above by "$\leq$" signs to show that firm 1's price distribution function first order stochastically dominates firm 2's. □

Proof of Proposition 2:

Using the symmetry property, the condition $c \geq l(1)$ can be written as $c \geq 1-h(0)$. This is equivalent to $\nu \geq 1$, as can be confirmed using (9).[76] To show that firm 1 conducts limit pricing, it must be shown that $F_1(p) \to 1 \;\; \forall p > c$ when $n \to 1$, so firm 1 chooses $p = c$ with probability one. Using (4), (12), and the symmetry property, one obtains (note, that since $\nu \geq 1$, case (ii) in (6) is always relevant):

$$F_1(p) = \frac{h(1-n)-(1-c)\big(1-h(n)\big)/(p-c)}{h(1-n)+h(n)-1} \tag{43}$$

$$\Rightarrow \;\; \lim_{n\to 1} F_1(p) = 1 - \frac{1-c}{p-c} \cdot \lim_{n \to 1}\big(1-h(n)\big)\big/h(0) \tag{44}$$

Since $h(n) \to 1$ for $n \to 1$, $\lim_{n\to 1} F_1(p) = 1 \quad \forall p > c$.[77] If $n = 1$, firm 2 may, instead of randomizing over $[c,1]$, simply choose $p = c$ with probability one as well. To see that this is an equilibrium (in pure strategies), note that firm 2 does not benefit from deviating to some higher price, because being the high-priced firm (without a customer base), its demand equals zero. If firm 1 deviates to some price greater than c, its demand drops to $l(1) < 1$. The best deviation is to choose the monopoly price, which yields a profit of $l(1)$. If nobody deviates, firm 1's profit equals c. Therefore, deviating is not profitable if $c \geq l(1)$. □

Proof of Proposition 3:

The first claim follows directly from (19) and (21). The second claim can be proved as follows: Using (20) and (21), and $\rho = 0$, one obtains the following equation that implicitly defines the function $G(.)$:

$$\frac{1}{p} = 1 + 2(M-1)\cdot G(p) + \frac{1-\lambda}{\lambda} M^2 \cdot G(p)^{M-1} \tag{45}$$

Since the left-hand-side (LHS) of this equation is greater than zero, finite $\forall p \in (0,1]$, and independent of M, while the RHS converges to infinity for $M \to \infty$ unless $G(p)$ converges to zero, $G(p)$ must converge to zero $\forall p \in (0,1]$ (and $F(p) \to 1$). Therefore, firms choose the competitive price (zero) with probability one. □

Proof of Proposition 4:

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[76] Note, that $h(1) = 1$ under the assumptions of Lemma 1, and see also the Proof of Lemma 4.

[77] The mixed strategy equilibrium, thus, converges to an equilibrium where firm 1 chooses $p = c$ with probability one, while firm 2 randomizes over the interval $[c,1]$.

The first claim follows from (19) and (21). The second claim is proved as follows: Using (20), (21), and $\lambda = 0$, one obtains the following equation that defines $G(.)$:

$$\rho = p\left(\rho + (1-\rho)M \cdot G(p)^{M-1}\right) \tag{46}$$

Unless $G(p)$ converges to one for $M \to \infty$ $\forall p \in [0,1)$, the RHS will converge to $p\rho$, which is smaller than the LHS (ρ) $\forall p \in [0,1)$. Therefore, $G(p)$ must converge to one for $M \to \infty$ $\forall p \in [0,1)$ (and $F(p) \to 0$), so firms choose the monopoly price 1 with probability one. □

Computation of expected loss, Section 2.4:

The computation of L as defined in (16) is shown. Let $Z \equiv P_1 - P_2$ be the random variable that describes the price differential, and let $f_Z(z)$ be the density function of Z.[78] (16), thus, reads:

$$L = n_1^2 \cdot \Pr\left[Z > 0\right] \cdot E\left[Z \mid z > 0\right] + n_2^2 \cdot \Pr\left[Z < 0\right] \cdot E\left[-Z \mid z < 0\right] \tag{47}$$

(47) can be written as:

$$L = n_1^2 \cdot E_{Z>0}\left[Z\right] - n_2^2 \cdot E_{Z<0}\left[Z\right] \tag{48}$$

, where $E_{Z>0}\left[Z\right] \equiv \int_0^{r-\underline{p}} z \cdot f_Z\left(z\right) dz$ and $E_{Z<0}\left[Z\right] \equiv \int_{-(r-\underline{p})}^{0} z \cdot f_Z\left(z\right) dz$.

To evaluate (48), $f_Z(z)$ must be determined. To this end, the density functions $f_i(p) \equiv F_i{}'(p)$, $i = 1,2$ are needed. Using (4) and (5), it is convenient to rewrite $F_1(p)$ and $F_2(p)$ as follows:

$$F_1(p) = \frac{1-\mu}{1-\underline{p}}\left(1 - \underline{p}/p\right), F_2(p) = \frac{1}{1-\underline{p}}\left(1 - \underline{p}/p\right) \tag{49}$$

, where $\mu \equiv 1 - F_1(1)$ is the probability mass that firm 1 attaches to the monopoly price. μ and $\underline{p}$ depend on $l(n)$, $l(1-n)$, $h(n)$, and $h(1-n)$. Using the symmetry property, one obtains:

$$\mu = 1 - \frac{h(1-n)}{h(n)}, \underline{p} = \frac{1-h(1-n)}{h(n)} \tag{50}$$

Using (49), one obtains for the density functions:

$$f_1(p) = \beta_1 / p^2 \text{ and } f_2(p) = \beta_2 / p^2 \tag{51}$$

, where $\beta_1 \equiv (1-\mu)\underline{p}/(1-\underline{p})$ and $\beta_2 \equiv \underline{p}/(1-\underline{p})$.

[78] Capitals are used for random variables, lower case letters for their realizations.

Since P_1 and P_2 are independent, $f_Z(z)$ can be written as a convolution integral:

$$f_Z(z) = \int_{-\infty}^{\infty} f_2\left(z+t\right) dF_1\left(t\right) \tag{52}$$

Evaluating (52), two cases must be distinguished: $z \geq 0$, and $z < 0$. Note also that, since firm 1's price distribution function has a mass point at $p = 1$, $f_1(p)$ is non-negative for $p \in [\underline{p}, 1]$, and infinite at $p = 1 + \varepsilon$.[79] Using (51), one, thus, obtains for $f_Z(z)$:

$$f_Z(z) = \begin{cases} \beta_1 \beta_2 \int_{\underline{p}}^{1-z} \frac{dt}{t^2 (z+t)^2} & \text{for } z \geq 0 \\ \beta_1 \beta_2 \int_{\underline{p}-z}^{1} \frac{dt}{t^2 (z+t)^2} + \frac{\beta_2 \mu}{(z+1)^2} & \text{for } z < 0 \end{cases} \tag{53}$$

Use (53) in (48) to obtain an expression for L. The integrals can be evaluated with the help of a computer. Then replace β_1, β_2, μ, and $\underline{p}$ to obtain L as a function of n, $h(n)$, and $h(1-n)$. Use (14) to replace $h(n)$ and $h(1-n)$ to obtain L as a function of n and λ (or, alternatively, as a function of n and ρ when word-of-mouth is prohibitively costly, so instead of λ, ρ is endogenized). This function is complicated, and the equation $L(n, \lambda) = z$ can not be solved for the endogenous value of λ analytically. But the function can be plotted, and the equation $L(n, \lambda) = z$ can be solved for λ numerically on a computer (see main text).

[79] Remember, that $F_i(p) \equiv \Pr[P_i < p]$. Therefore, the jump in $F_1(.)$ lies marginally above 1.

Chapter 3. Dynamic Price Competition under Word-of-Mouth Communication

3.1. INTRODUCTION

The model introduced in this chapter extends the analysis of the previous chapter to an infinitely repeated pricing game between duopolistic firms in a homogeneous goods market. Whereas in chapter 2, the focus is on the shape of the past sales – current profit relation, the focus of the present chapter is on *market share dynamics*. As in the static version of the model, firms use mixed pricing strategies in equilibrium, and the resulting price dispersion generates market share dynamics. Note, that since there are no external sources of uncertainty or noise in the model, the dynamics result solely from firms' profit maximizing behavior.

The prediction that mixed pricing strategies lead to price dispersion in equilibrium is well-known from the literature on consumers' search (see e.g. Varian, 1980, and Stahl, 1989). There is also empirical evidence that price dispersion plays an important role in real-world markets, and that it persists over time so that consumers can not learn which store is the low-priced one (Lach, 2002). However, authors in the theoretical seach literature have paid little attention to the market share dynamics that result naturally from such price dispersion. In the absence of any intertemporal links that emerge when consumers are acting as repeat purchasers, the dynamics of a search market can be represented by a simple repetition of a static game.

Chen and Rosenthal (1996) introduce an infinitely repeated pricing game with an explicit intertemporal link. The authors assume that the firm that currently offers the lower price loses market shares compared to the previous period, but its market share does not drop to zero (unless it was zero or just above zero before). However, the authors assume that a firm that looses or gains market shares always looses or gains exactly the same number of consumers. There is, thus, a uniform 'step size' upwards and downwards in the market share space, an assumption that seems poorly justified. This chapter offers a more rigorous microeconomic foundation for market share dynamics generated by mixed pricing equilibria.

The underlying information process is the same as the one introduced in chapter 2. The main assumptions are briefly summarized here for convenience. By assumption, all consumers have made a purchase in this market before, and remember the relevant information[80] about their previous supplier.[81] Consumers who become informed about both firms' offers purchase the cheaper one (if prices are identical, they return to their previous supplier). The other consumers always return to their previous supplier and buy unless the price exceeds the reservation price. There are three types of consumers. Let λ be the fraction of word-of-mouth consumers in the population. These consumers ask one other consumer randomly chosen from the population about this consumer's previous choice, and in case it differs from the consumer's own previous choice, the consumer learns the relevant information about the other firm. Let ρ be the fraction of ignorant types. These consumers perform neither search nor word-of-mouth communication, and simply return to their previous supplier. The fraction of searchers equals $(1-\lambda-\rho)$. Searchers perform active search and are informed about both firms' current offers. By assumption, λ and ρ are exogenously fixed, and the customer base of both firms contains the same fraction of word-of-mouth consumers and ignorant types in every period (irrespective of the history of the game). The 'type' of a consumer is, thus, a transient attribute.

Apart from the above assumptions on the information process, an important feature of the model introduced in this chapter is that I allow for different frequencies of repeat purchasing. The model covers the extreme cases where all consumers make a purchase in each period (non-durable goods), and where the frequency of purchases per consumer approaches zero (durable goods). If the frequency of repeat purchasing is low, current market demand is more volatile than the total size of a firm's customer base, including all consumers who purchased the firm's product when they last made a purchase in this market. This adds another source of inertia to the model, and may explain why firms are often so concerned about market share targets. It also helps to explain why entry into a market with incomplete consumer information may be rather time consuming: even if the entrant undercuts the incumbent's price for a sustained period of time, its market share increases only gradually.

In order to disentangle effects that stem from firms' profit maximizing behavior, and effects that stem from the properties inherent to the information process, the analysis of market share dynamics is split into several subcases. The first case (the "*stochastic benchmark case*")

[80] The 'relevant information' concerns the existence and location of the respective supplier, and knowledge about the product's characteristics. Once this information is obtained, a current price quote can be obtained for free.

[81] Consumers, thus, forget the information from earlier consumption experiences. An alternative interpretation is that old consumers are replaced by young consumers who enter the market and inherit information from their parents, but only about the product that the parents consumed previously.

considers the dynamic evolution of market shares in the absence of strategic interaction. This allows to isolate the driving forces behind the dynamics that are inherent to the information process. Under the stochastic benchmark, the probability that each of the firms charges the lower price in the market is, by assumption, equal to 1/2 in each period. Since analytical results are surprisingly difficult to obtain (the reasons for this will become clear in the main text), the analysis is supplemented by a "deterministic benchmark". Under this case, firms simply alternate in choosing the higher and the lower price in the market. Using a fixed point analysis, analytical results are derived that help to describe and explain the patterns of market share evolution in the stochastic benchmark. In brief, the stochastic benchmark produces highly volatile market share dynamics if most consumers are searchers ($\lambda + \rho$ near zero). Market shares, then, oscillate between the extremes. If the fraction of ignorant types in the population, ρ, is increased, market shares become less volatile, and the market split tends to be an even one most of the time. If the fraction of word-of-mouth consumers, λ, is large, the market split tends to be a skewed one, but unlike in the case where most consumers are fully informed, a firm that obtains a large market share in some period tends to maintain the dominant position in the market for many consecutive periods, which can be attributed to the popularity weighting property of the word-of-mouth process.

The second case (the "*repeated static game*") considers the dynamic evolution of market shares under firms' strategic interaction when future profits are fully discounted. Firms' incentives to invest (or disinvest, depending on the shape of the value function) in future market shares are, thus, "switched off". This allows to isolate the dynamic effects stemming from firms' incentives to maximize current expected profits. To this end, the resulting market share dynamics are compared with the stochastic benchmark case. The main result is that, unless most consumers are searchers, market splits tend to be more even than under the stochastic benchmark. This is because a firm with a smaller customer base prices more aggressively than a firm with a large customer base and, thus, tends to gain market shares compared to the previous period.

The third case is the *full-fledged dynamic model* where future profits are not fully discounted. The driving forces isolated in the stochastic benchmark and in the repeated static case are still present, but supplemented by firms' incentives to invest or disinvest in future market shares. The main results are as follows. If most consumers are searchers, market shares are very volatile, and the low-priced firm serves most of the current market demand irrespective of the size of its customer base. The incentives to invest in future market shares are, thus, almost negligible, and market share dynamics closely resemble those under the stochastic benchmark

and the repeated static case. If λ is large, the market split tends to be a more skewed one than in the repeated static game. This result can be attributed to the fact that a firm with a large customer base has an incentive to *defend* its customer base against an aggressive competitor with a smaller customer base, an effect that is not present in the repeated static case. This counteracts the tendency towards even market splits observed in the repeated static case.
The rest of the chapter is organized as follows. In Section 3.2, the basic model is introduced, and equilibrium conditions are derived. The analysis is performed on a general level and is not restricted to the information process introduced above. Section 3.3 discusses market share dynamics under the above information process in the absence of strategic interaction. (Section 3.3.1 deals with the deterministic, and Section 3.3.2 with the stochastic benchmark case.) Section 4 discusses market share dynamics under firms' strategic interaction. (Section 3.4.1 analyzes the repeated static case, and Section 3.4.2 deals with the full-fledged dynamic game). Section 3.5 concludes. Some proofs are relegated to the Appendix.

3.2. THE MODEL

There are two firms ($i \in \{1,2\}$) in a market for a homogeneous good. Time is discrete, the horizon is infinite, and firms simultaneously choose prices $p_{i,t}$ in every period t ($t = 1,2,...$). The firms' marginal costs are constant and identical and, for simplicity, normalized to zero. On the demand side, there is a continuum of consumers with measure Ω. Consumers are repeat purchasers, and all consumers have made a purchase in this market before. Each marginal consumer desires to have exactly one unit of the product in each period, and the valuation per period is 1 for all consumers. Therefore, the monopoly price in this market equals 1, and prices above 1 can be eliminated from the firms' strategy space without loss of generality.[82] It is assumed that the good is durable, albeit with a limited durability. Let the failure rate per period be $\alpha \le 1$. The mass of consumers who make a purchase per period is, thus, $\alpha \cdot \Omega$. By appropriately adjusting the measure of the population size, we can assume that a unit mass of consumers make a purchase each period:

$$\alpha \cdot \Omega = 1 \tag{1}$$

Firm i's demand ($i \in \{1,2\}$) in period t is denoted by $D_{i,t}$. Under the above assumptions, we have $\forall t \ge 1$:

$$D_{1,t} + D_{2,t} = 1 \tag{2}$$

[82] Prices below zero are possible. Since marginal costs are normalized to zero, they are interpreted as sales below marginal cost.

Note, that firm i's realized market share in period t is identical to firm i's demand $D_{i,t}$.

By assumption, firm i's demand in period t may depend upon prices in period t, as well as upon the distribution of the firms' market shares in previous periods. Let Ω_t be the mass of consumers who purchased firm 1's product when they last made a purchase in this market, evaluated at the beginning of period t. Among these consumers, let n_t be the mass of those who make a *new* purchase in period t. I will refer to n_t as the size of firm 1's *customer base* in period t. Under the above assumptions, we have:

$$n_t = \alpha \cdot \Omega_t \tag{3}$$

The size of firm 1's customer base, n_t, is used as the state variable in period t.

The law of motion for Ω_t is given by the following equation:

$$\Omega_t = (1-\alpha)\Omega_{t-1} + D_{1,t-1} \tag{4}$$

Using (3), (4) can be written as:

$$n_t = (1-\alpha)n_{t-1} + \alpha D_{1,t-1} \tag{5}$$

Note, that, if the frequency of purchases α equals 1, then the size of firm 1's customer base in period t is equal to firm 1's demand in the previous period.

By assumption, firm 1's current demand may only depend on whether it charges the higher or the lower price in the current period, and on the size of its customer base. Let $l(n)$ be firm 1's demand if it charges the higher price in the market, given that the current size of its customer base is n. Similarly, let $h(n)$ be firm 1's demand if it is the low-priced firm. If both firms charge an identical price, consumers purchase from their previous supplier. By assumption, the functions $l(.)$ and $h(.)$ are time-invariant and identical for both firms. Since the demand of the high- and low-priced firm depends upon the amount of information available to the consumers, a specification of $l(.)$ and $h(.)$ is called an *information process*.

Given some information process $(l(.), h(.))$, if the current state (the size of firm 1's customer base) is n, and current prices are p_1 and p_2, firm 1's current demand, thus, equals:[83]

$$D_1(n, p_1 \mid p_2) = \begin{cases} h(n) & \text{if } p_1 < p_2 \\ n & \text{if } p_1 = p_2 \\ l(n) & \text{if } p_1 > p_2 \end{cases} \tag{6}$$

[83] Time subscripts are omitted when this does not lead to confusion.

(1) defines a generalized Bertrand model where, due to incomplete information on the consumer side, the firm that charges the lower price does not serve the entire market demand. Firm 1's expected profit in the current period is $\pi_1^E(n,p_1) \equiv p_1 E_{p_2}\left[D_1(n,p_1 \mid p_2)\right]$. Firm 2's demand and profit are obtained by replacing n by $1-n$.

Under the above assumptions, the following symmetry property holds for any information process $\left(l(.),h(.)\right)$ (it follows from (2) and (1)):

$$l(n) = 1 - h(1-n) \quad \forall n \in [0,1] \tag{7}$$

By assumption, $h(.)$ is continuous, strictly increasing, and $n < h(n) < 1$ holds $\forall n \in [0,1)$.

Given some information process $\left(l(.),h(.)\right)$, the history of market shares up to period t is fully described by the initial size of firm 1's customer base in period 1 (n_1), and a binary sequence $s_t \equiv \{I_\tau\}$ ($\tau = 1,2,...,t$), with $I_\tau = 1$ if $p_{1,\tau} < p_{2,\tau}$, and $I_\tau = 0$ otherwise.[84] If firm 1 *gains* market shares in period τ, the τ'th entry in the sequence is, thus, equal to 1.

Let δ be the discount factor for future profits.[85] In period t, firm i maximizes the present discounted value of future expected profits over an infinite number of periods:

$$V_i(H_t) \equiv \max_{p_{i,t},p_{i,t+1},...} E\sum_{\tau=t}^{\infty} \delta^{\tau-t}\pi_{i,\tau}^E(p_{i,\tau} \mid H_\tau) \tag{8}$$

, where H_t denotes the history up to period t. The history H_t contains all prices chosen up to period $t-1$, as well as the initial state n_1.[86] The expectation in (8) is over firm j's prices ($j \neq i$). Note, that there is no external source of uncertainty in the model. In equilibrium, uncertainty arises only if firms use *mixed pricing strategies*.

In general, firms can condition their choice in period t on the entire history up to period t, H_t. However, as is well-known from the Folk-theorem, the set of possible equilibria in dynamic pricing games is potentially overwhelming. In order to rule out collusive equilibria, a useful equilibrium refinement appears to be *Markov perfection*. The payoff-relevant state variable in period t is n_t. Under Markov perfection, firms are restricted to condition their price only on the current state.

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[84] The event $p_{1,\tau} = p_{2,\tau}$ occurs with probability zero in equilibrium and can be neglected.

[85] The case where firms are myopic corresponds to $\delta = 0$. Note, that already this case produces non-trivial market share dynamics (see Section 3.4.1).

[86] The states in periods 2 through t, captured by the sequence s_t that describes firm 1's market share gains and losses, can be inferred from the sequence of prices and n_1.

The Bellman-equation for a firm with a customer base of size n (firm 1) reads:

$$V(n) = \max_p \left[\pi^E(n,p) + \delta E\left[V(n') \mid n,p\right]\right] \tag{9}$$

, where n and n' stand for n_t and n_{t+1}, respectively. By omitting the subscript for the identity of the firm, (9) contains the following symmetry assumption: a firm's present value only depends upon the size of its current customer base, n, and not upon the identity of the firm. The value of the competitor (firm 2) is, thus, obtained by replacing n by $1-n$.

Lemma 1: If δ is sufficiently small, there is no Markov perfect equilibrium that comprises pure strategies for any $n \in [0,1]$.

The result in Lemma 1 is restricted to the case where the discount factor for future profits, δ, is "sufficiently small". This is because an increase in the size of a firm's customer base may lead to *reduced* profits in the future as the intensity of future price competition may increase. If δ is large, a firm may, thus, be reluctant to undercut the competitor's price if this price can be predicted with certainty.

Let the equilibrium price distribution function of a firm with a customer base of size n in the present period be $F(n,p)$, with the convention that the mass or density at the price p itself is not included, that is: $F(n,p) \equiv \Pr[P < p \mid n]$, where P is the random variable from which p is drawn.[87] The price distribution function $F(n,p)$ does not depend on the identity of the firm (index i) because it is implicitly assumed that the equilibrium is symmetric in the sense that, exchanging n by $1-n$, the roles of the firms are inverted, but the equilibrium is otherwise not affected.

Lemma 2: If δ is sufficiently small, $F(n,p)$ and $F(1-n,p)$ have the same support[88]. The support is convex in p $\forall n \in [0,1]$, the maximum is the monopoly price 1, and the minimum (denoted by $\underline{p}$) is smaller than 1.

Lemma 3: If δ is sufficiently small, there is no Markov perfect equilibrium where more than one firm attaches positive probability mass to any single price. Furthermore, if a mass point is part of a firm's equilibrium strategy for the given state n, it is located at the monopoly price.

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[87] Under this convention, $1-F(n,1)$ is the probability mass at the monopoly price.

[88] The support is the smallest closed set of prices whose complement has a probability of zero.

Consider the profit maximization problem of a firm with a customer base of size n (firm 1). Suppose, the competitor (firm 2) randomizes over its price according to the equilibrium distribution function $F(1-n,p)$. Firm 1's expected profit in the current period is given by:

$$\pi^{E}(n,p)=p\left(l(n)F(1-n,p)+h(n)\left(1-F(1-n,p)\right)\right) \tag{10}$$

, where $l(n)F(1-n,p)$ is firm 1's demand if it chooses the higher price in the market times the probability of this event.

Firm 1's expected value in the next period, given that it chooses p this period, is given by:

$$E\left[V(n')\,|\,n,p\right]=V\left(l(n)\right)F(1-n,p)+V\left(h(n)\right)\left(1-F(1-n,p)\right) \tag{11}$$

, where $V\left(l(n)\right)F(1-n,p)$ is the firm's value after losing market shares (the size of its customer base is, then, $l(n)$) times the probability that the competitor chooses the lower price.

In a mixed strategy equilibrium, the maximum in (9) is attained over a set of prices, and the support of $F(n,p)$ contains only prices within this set.[89] For all prices p outside the support of $F(n,p)$, it must hold that:

$$\pi^{E}(n,p)+\delta E\left[V(n')\,|\,n,p\right]\leq V(n) \tag{12}$$

(12) is to assure that there is no profitable deviation from the equilibrium strategy $F(n,p)$.

Using (10) and (11) in (9), we obtain the following expression for the value function $V(n)$:

$$\begin{aligned} V(n)&=ph(n)-p\left(h(n)-l(n)\right)F(1-n,p)\\ &\quad+\delta V\left(h(n)\right)-\delta\left(V\left(h(n)\right)-V\left(l(n)\right)\right)F(1-n,p) \end{aligned} \tag{13}$$

The $\max$ - operator has been omitted because, for prices within the support of $F(n,p)$, the right-hand side must be independent of p in a mixed strategy equilibrium.

(13) can be solved for the price distribution function $F(1-n,p)$. Replacing $1-n$ by n, we obtain for the price distribution function of a firm with a customer base of size n:

$$F(n,p)=\frac{\delta V\left(h(1-n)\right)-V\left(1-n\right)+h(1-n)p}{\delta V\left(h(1-n)\right)-\delta V\left(l(1-n)\right)+\left(h(1-n)-l(1-n)\right)p}\ ,\ \forall n\in[0,1] \tag{14}$$

If the value function $V(.)$ is known, (14) allows to compute the price distribution function for all possible values of the state, n. For a fixed value of n, $F(n,p)$ depends upon the value of the $V(.)$ - function only at the positions $1-n$, $h(1-n)$, and $l(1-n)$.

[89] Otherwise, the firm would choose prices that yield a lower expected profit than the prices within the set with positive probability, so this can not be an equilibrium.

Equilibrium conditions:

By Lemma 2, the following condition must be fulfilled in equilibrium:

$$\exists\, \underline{p}:\ F(n,\underline{p}) = 0\ \wedge\ F(1-n,\underline{p}) = 0 \tag{15}$$

Using (14), (15) yields:

$$\underline{p} = \frac{V\left(1-n\right) - \delta V\left(h(1-n)\right)}{h(1-n)}\ \text{, and}\ \ \underline{p} = \frac{V\left(n\right) - \delta V\left(h(n)\right)}{h(n)} \tag{16}$$

Equalizing the two expressions for $\underline{p}$ in (16), we obtain the first equilibrium condition:

$$h(n)V\left(1-n\right) - \delta h(n)V\left(h(1-n)\right) = h(1-n)V\left(n\right) - \delta h(1-n)V\left(h(n)\right) \tag{17}$$

The second equilibrium condition follows from Lemma 3:

$$\left(\text{(a)}\ F(n,1) \le 1\ \wedge\ F(1-n,1) = 1\right)\ \vee\ \left(\text{(b)}\ F(n,1) = 1\ \wedge\ F(1-n,1) < 1\right) \tag{18}$$

According to (18), the maximum of the support of $F(n,p)$ and $F(1-n,p)$ is the monopoly price 1, and at most one of the two price distribution functions can have a mass point at $p = 1$. The first case in (18) is referred to as “case a”, the second as “case b”. Under case a, the price distribution function of the firm with the customer base of size n (firm 1) can have a mass point at the monopoly price, and the competitor’s price distribution function can not (vice versa for case b). Intuitively, we would expect that firm 1 chooses the monopoly price with positive probability whenever its customer base is greater than 1/2 (case a), because it should have a larger incentive to exploit its monopoly power over uninformed repeat purchasers, while firm 2 has a greater incentive to increase the size of its customer base and, thus, prices more aggressively. If firm 2’s customer base is greater than 1/2, we expect firm 2’s price distribution function to have a mass point at $p = 1$ (case b).

Suppose, for the given state n, case a is relevant. By (18), the condition $F(1-n,1) = 1$ must hold in equilibrium. Using (14), this can be written as:

$$V\left(n\right) - \delta V\left(l(n)\right) = l(n) \tag{19}$$

If case b is relevant (given n), using $F(n,1) = 1$ in (14), we obtain:

$$V\left(1-n\right) - \delta V\left(l(1-n)\right) = l(1-n) \tag{20}$$

Together with an (until now) unknown rule that states whether case a or case b is relevant for every possible state n, (17), (19), and (20) implicitly define the value function $V(.)$. Note, that this is a continuum of equations because (17) and (19), resp. (20), must be fulfilled $\forall n \in [0,1]$.

Solution procedure:

There are (at least) two possible solution procedures. One is based on a *discretization* of the state space ($n \in [0,1]$). The equilibrium conditions (17) and (19), resp. (20), can, then, be solved analytically with the help of a computer using the market share grid. Note, that this is simply a system of linear equations. The only difficulty is that the rule that determines when case a and case b is relevant (that is, whether (19) or (20) must be used for a given n) is not known. As mentioned above, the most obvious guess is that case a is relevant if $n \geq 1/2$, and case b otherwise. Using this guess, a candidate equilibrium can be computed. It must, then, be checked that condition (18) is fulfilled for all values of n. If it is not fulfilled, then the decision rule must have been wrong, or no equilibrium exists for the given parameter set. It turns out that, if δ is sufficiently small, the above decision rule is always correct, and the solution procedure yields good results if the market share grid is sufficiently fine. Problems arise only if the parameters λ and δ are both fairly large (see below). For large values of ρ and δ, a solution is always obtained. The difficulties may, thus, be related to feedback effects due to the popularity weighting property of the word-of-mouth process.

The second procedure is to *iterate* on the value function. While the first procedure (discretizing the state space) yields an analytical solution to an approximated problem, the iteration procedure yields an approximate solution to the original problem. The iteration procedure has the advantage that the decision rule to be used at each iteration round, that states whether case a or b is relevant for any given n, can be derived analytically. The procedure is now described in more detail.[90]

The iteration process is similar to a solution by backwards induction of a finite game with T periods. The first approximation of the value function $V(.)$ that is used to start the iteration is $\tilde{V}(n) \equiv 0 \quad \forall n \in [0,1]$ ($\tilde{V}$ denotes iterated value function). Counting backwards, the iterated value function for period $t < T$ in the hypothetical finite game is computed recursively. Replacing V by $\tilde{V}$, the equilibrium conditions (17), (19), and (20) read after rearranging:[91]

$$\tilde{V}(n) = \frac{h(n)}{h(1-n)}\left(\tilde{V}(1-n) - \delta\tilde{V}'\left(h(1-n)\right)\right) + \delta\tilde{V}'\left(h(n)\right) \tag{21}$$

[90] It is sometimes useful to apply both procedures when complications arise. This helps to find out whether the complications are due to the solution procedure (or its implementation on a computer), or due to the fact that the dynamic program does not have a (stable) solution for the given parameter values (see below).

[91] Furthermore, taking account of the iteration procedure, $\tilde{V}$ and $\tilde{V}'$ is to be distinguished (they are not identical unless the iteration process has converged).

$$\tilde{V}\left(n\right)=l(n)+\delta\tilde{V}'\left(l(n)\right) \tag{22}$$

$$\tilde{V}\left(1-n\right)=l(1-n)+\delta\tilde{V}'\left(l(1-n)\right) \tag{23}$$

(22) is valid if case a is relevant. If this holds for the given value of n, then the first part of condition (18) must be fulfilled, too ((22) was derived from the second part). That is, it must hold that $F(n,1)\leq 1$. Using (21) and (22) in (14), this yields the following condition that assures that case a is relevant:

$$\frac{h(1-n)-\dfrac{h(1-n)}{h(n)}\left(l(n)+\delta\tilde{V}'\left(l(n)\right)-\delta\tilde{V}'\left(h(n)\right)\right)}{\delta\tilde{V}'\left(h(1-n)\right)-\delta\tilde{V}'\left(l(1-n)\right)+\left(h(1-n)-l(1-n)\right)}\leq 1 \tag{24}$$

If (24) is fulfilled, case a is relevant, if it is violated, case b is relevant. Using the definition:

$$\beta(n)\equiv h(1-n)\left(l(n)+\delta\tilde{V}'\left(l(n)\right)-\delta\tilde{V}'\left(h(n)\right)\right) \tag{25}$$

, (24) can be written more conveniently as follows:

$$\beta(n)\geq\beta(1-n) \tag{26}$$

If (26) is fulfilled, then case a is relevant for the given value of n. Obviously, (26) holds with equality for $n=1/2$. Therefore, at this point, there is always a potential change from case a to case b (or vice versa). For all values of n that fulfill (26), the iteration formula (22) is to be used in the current iteration round. For all other values of n, case b is relevant. Plugging the expression for $\tilde{V}\left(1-n\right)$ in (23) into (21), we obtain the following iteration formula for case b:

$$\tilde{V}\left(n\right)=\frac{h(n)}{h(1-n)}\left(l(1-n)+\delta V'\left(l(1-n)\right)-\delta\tilde{V}'\left(h(1-n)\right)\right)+\delta\tilde{V}'\left(h(n)\right) \tag{27}$$

Using the definition of β, this can be written more conveniently as:

$$\tilde{V}\left(n\right)=\frac{\beta(1-n)}{h(1-n)}+\delta\tilde{V}'\left(h(n)\right) \tag{28}$$

Combining (22), (26), and (28), the complete iteration formula for both cases reads:

$$\tilde{V}(n)=\begin{cases} l(n)+\delta\tilde{V}'\left(l(n)\right) & \text{if } \beta(n)\geq\beta(1-n) \\ \dfrac{\beta(1-n)}{h(1-n)}+\delta\tilde{V}'\left(h(n)\right) & \text{if } \beta(n)<\beta(1-n) \end{cases} \tag{29}$$

Using (29), the iteration on the value function $V(.)$ is easy to implement on a computer.

Existence of equilibrium

For small values of δ, the iteration procedure converges quickly. If λ and δ are both fairly large, the iteration process often takes many rounds to converge or fails to converge.[92] As a general rule, I have found that the iteration process converges whenever (26) is fulfilled with equality only at $n = 1/2$ (and no other value of n) after several iteration rounds, which corresponds to the postulated simple decision rule mentioned above. The iteration procedure, and the procedure with the discretized state space, then, yield identical results up to some numerical imprecision. Apparently, the lack of a solution (if it occurs) is related to highly irregular patterns in the shape of the invariant distributions (see Section 3.4). It turns out that a small change in the initial state n_1 may, then, affect the probability distribution of the firms' customer base sizes over many future periods and can, thus, lead to a substantial change in a firm's value if the discount factor for future profits, δ, is sufficiently large. In such situations, the dynamic program may not have a solution. The observation that, for some parameter values, a small alteration in the starting value n_1 can lead to a significantly different evolution of market shares over many periods is reminiscent of chaotic behavior.

3.3. MARKET SHARE DYNAMICS IN THE ABSENCE OF STRATEGIC INTERACTION

Before analyzing market share dynamics generated by competition among profit maximizing firms, it is useful to characterize these dynamics in a *non-strategic* environment, given the word-of-mouth information process introduced in Section 3.1. Consider as a benchmark the simple stochastic process that assigns equal probabilities of winning or losing market shares to both firms in each period. It turns out that this benchmark process without strategic interaction already generates non-trivial market share patterns. The dynamic evolution of market shares in a *strategic* environment can only be understood against the background of the dynamic properties inherent to the information process, that are revealed by the stochastic benchmark case.

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[92] Whether this is the case also depends on the failure rate α. If α is small, the value function converges also for large values of δ when λ is large. The problem becomes more "nicely behaved", which is apparently related to the fact that the invariant distributions (of market shares and of the state) are less irregular for these parameter values. When λ is small, the situation is reversed. For $\alpha = 1$, the iteration, then, converges for all values of δ, while for small values of α, the invariant distributions become highly irregular, and the iteration does not converge if δ is large. See below.

Under the word-of-mouth information process introduced in Section 3.1, the mass of consumers in firm 1's customer base who find out about firm 2's current offer (given that the current state is n) equals:

$$(1-\rho)n-\lambda n^2 \tag{30}$$

, where λn^2 is the mass of consumers in firm 1's customer base who rely on word-of-mouth and ask a consumer within firm 1's customer base.

Using (13), we obtain for the functions $l(.)$ and $h(.)$ introduced in Section 3.2:

$$l(n)=\rho n+\lambda n^2 \;,\;\; h(n)=1-\rho(1-n)-\lambda(1-n)^2 \tag{31}$$

It is easy to verify that this information process fulfills the assumptions made in Section 3.2 if $0<\lambda+\rho$ and $\rho<1$ hold.

To get an idea why the information process specified in (14) can lead to non-trivial market share dynamics, consider a simple numerical example. Suppose, the initial size of firm 1's customer base is $n_1=0.5$, and let $\alpha=1$, $\lambda=0$, and $\rho=0.6$. If firm 1 loses market shares in period 1, and gains market shares in period 2, the sequence of firm 1's market shares is: $n_2=0.2$, $n_3=0.6$. As the example illustrates, a loss of market shares followed by a gain of market shares does usually not yield the initial market share again ($n_3\neq n_1$).

More generally, the size-distribution of firms' customer bases in period t not only depends upon the initial market split n_1 and the number of periods with market share gains and losses, but on the *exact sequence* of events. The non-trivial market share dynamics 'inherent' to the information process stem from the fact that the step sizes "upwards": $h(n)-n$, and "downwards": $n-l(n)$ are usually not identical.

As mentioned before, the benchmark case against which to compare sequences generated under strategic pricing behavior is the stochastic process assigning equal probabilities of gaining or losing market shares to both firms in all periods: $\Pr[p_{1,t}<p_{2,t}]=1/2\ \ \forall t\geq 1$. To obtain tractable results, I first analyze the evolution of market shares under the following *deterministic* sequence of events, that is used as a proxy for the stochastic benchmark: $S^{alt}\equiv(0,1,0,1,...)$. Under this sequence of events, firms alternate in setting the higher and the lower price in the market. The idea is that, if market shares converge to 'fix points' under this alternating sequence (the "deterministic benchmark"), then the stochastic benchmark will produce markets shares *near* these fix points with high frequency. The analysis of the deterministic benchmark, thus, yields some qualitative predictions about the behavior of the

stochastic benchmark, and it helps to get an intuition of the main driving forces underlying the stochastic market share patterns.

3.3.1. The deterministic benchmark

For simplicity, the analysis in this Section is restricted to the case $\alpha = 1$ (each consumer makes a purchase in each period). To characterize the evolution of market shares when firms alternate in setting the higher and the lower price, it is useful to define *fix points*:

Definition 1: Given an information process $\left(l(.), h(.)\right)$, n^{fix} is a fix point of the sequence of firm 1's market shares if $n^{fix} = h\left(l(n^{fix})\right) \in [0,1]$.

According to this definition, if firm 1 starts from a fix point n^{fix}, then loses and afterwards gains market shares once, it reaches its original market share n^{fix} again. Therefore, the fix point is reached in all even-numbered periods under the alternating sequence S^{alt}. In all odd-numbered periods, the market share $l(n^{fix})$ is attained.

Fix points are points that the sequence of firm 1's market shares can either converge to or diverge away from. If it *converges* to a fix point, starting from any initial market share n_1 within some non-empty neighborhood of n^{fix}, the fix point is 'stable':

Definition 2: A fix point n^{fix} of an information process $\left(l(.), h(.)\right)$ is stable if there is a neighborhood $U_\varepsilon(n^{fix}) \equiv \left\{n \in [0,1]:\ \left|n - n^{fix}\right| < \varepsilon\right\}$, with $\varepsilon > 0$, such that every sequence of firm 1's market shares that starts from some n_1 within $U_\varepsilon(n^{fix})$ converges to n^{fix} under $S^{alt} = (0,1,0,1,...)$.

To simplify the exposition, it is useful to define the following function:

$$g(n) \equiv h\left(l(n)\right) \tag{32}$$

The sequence of firm 1's realized market shares in even-numbered periods, expressed in terms of the state n, is under the alternating sequence S^{alt} given by the following law of motion:

$$n_t = g(n_{t-2}) \tag{33}$$

This is a simple deterministic process that can be analyzed using standard tools of dynamic systems theory. According to Definition 1, a fix point fulfills:

$$n^{fix} = g(n^{fix}) \tag{34}$$

Lemma 4: Given some information process $\left(l(.),h(.)\right)$, if $g(.)$ is continuous, and n^{fix} is a fix point, then n^{fix} is stable if $\left|dg(n)/dn\right|_{n=n^{fix}} < 1$.

A proof is omitted since this is a standard result from dynamic systems theory.

Lemma 5: Given some information process $\left(l(.),h(.)\right)$, if n^{fix} is a fix point, then $h(1-n^{fix})$ is also a fix point.

Proof:

$h(1-n^{fix})$ is a fix point if $h(1-n^{fix}) = g(h(1-n^{fix})) = h\left(l\left(h(1-n^{fix})\right)\right)$ holds. By strict monotonicity of $h(.)$, this is equivalent to: $1-n^{fix} = l\left(h(1-n^{fix})\right)$. Applying the symmetry property $h(n) = 1-l(1-n)$ twice, this yields: $n^{fix} = h\left(l\left(n^{fix}\right)\right) = g(n^{fix})$, which is true since n^{fix} is a fix point.

Definition 3: Given some information process $\left(l(.),h(.)\right)$, a fix point n^{fix} is called *degenerate* if the corresponding fix point $h(1-n^{fix})$ is identical to n^{fix}.

Using the symmetry property, the condition $n^{fix} = h(1-n^{fix})$ can be written as $l(n^{fix}) = 1-n^{fix}$. Starting from a degenerate fix point $n^{fix} > 1/2$, the sequence of firm 1's market shares alternates between the values $l(n^{fix}) = 1-n^{fix} < 1/2$ and $n^{fix} > 1/2$ that are located symmetrically around $n = 1/2$. Non-degenerate fix points are generally located further away from the center of the market share space, and if $n^{fix} > 1/2$ holds, then $l(n^{fix}) > 1/2$.

For the information process specified in (14), using (34), we obtain the following fix points:[93]

[93] Note, that n^{*} and n^{**} are only fix point if they lie in the interval [0,1] for the given parameter values. There are two other solutions to (34). One lies always outside the interval [0,1], the other is the corresponding fix point to n^{**}.

$$n^* \equiv \left(\sqrt{(1+\rho)^2+4\lambda}-1-\rho\right)/2\lambda \ , \ n^{**} \equiv \left(\sqrt{(1+\rho)^2-4(1-\lambda)}+1-\rho\right)/2\lambda \tag{35}$$

It is straight-forward to show that n^* is degenerate, while n^{**} is non-degenerate. This is illustrated in Figure 1, for $\lambda = 0.7$ and $\rho = 0.2$, where n^{***} denotes the corresponding fix point to n^{**}: $n^{***} \equiv h(1-n^{**})$.

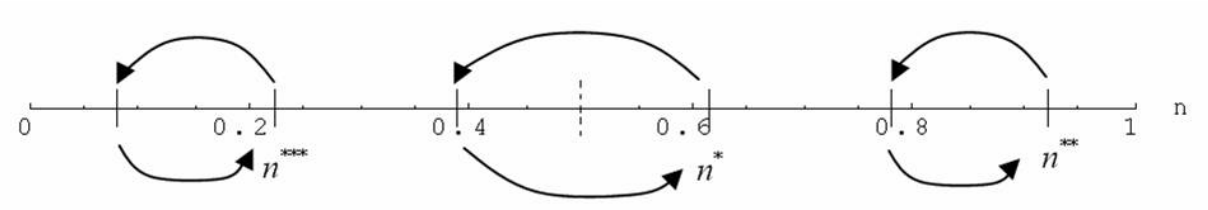


Figure 1: Fix points in the market share space; λ = 0.7, ρ = 0.2

To obtain tractable results, it is useful to analyze the following special cases separately: 1. $\rho = 0$, and 2. $\lambda = 0$.

<u>Case 1</u>:

Using $\rho = 0$, we obtain the following expression for $g(.)$ using (14) and (32):

$$g(n) = 1-\lambda\left(1-\lambda n^2\right)^2 \tag{36}$$

The expressions for n^* and n^{**} in (35) simplify to:

$$n^* \equiv \left(\sqrt{4\lambda+1}-1\right)/2\lambda \ , \ n^{**} \equiv \left(\sqrt{4\lambda-3}+1\right)/2\lambda \tag{37}$$

Figure 2 shows n^* and n^{**} as functions of λ, for $n \geq 1/2$.[94]

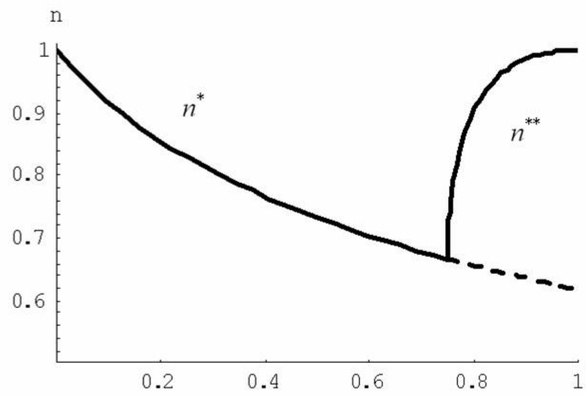


Figure 2: Fix points as a function of λ; ρ = 0.5 (dashed: unstable fix point)

As illustrated in Figure 2, $n^* \in [0,1]$ holds for the entire interval $\lambda \in [0,1]$. At $\lambda = 3/4$, there is a bifurcation where the fix point n^{**} starts to exist.

[94] Due to the symmetry property, the picture for $n \leq 1/2$ contains no additional information.

Proposition 1.1: Given the information process specified in (14), for $\rho = 0$, the fix point n^* is stable if $\lambda < 3/4$. n^{**} is stable if $\lambda > 3/4$.

Proof:

By Lemma 1, n^* (n^{**}) is stable if $\left|dg(n)/dn\right|_{n=n^*} < 1$ ($\left|dg(n)/dn\right|_{n=n^{**}} < 1$) holds. Using (36) and (37), this condition yields $\lambda < 3/4$ ($\lambda > 3/4$) as claimed.

It can be shown that for $\lambda < 3/4$ and $\rho = 0$, all sequences of market shares converge to n^* under the alternating sequence S^{alt} (irrespective of the starting point n_1), while for $\lambda > 3/4$, convergence to n^{**} is obtained if $n_1 > n^*$, and to n^{***} if $n_1 < n^*$.[95]

According to (37) and Proposition 1.1, for $\rho = 0$, we expect the stochastic benchmark process with $\Pr[p_{1,t} < p_{2,t}] = 1/2 \;\; \forall t > 0$ to yield market shares near the extremes when λ is small. The step-size upwards, following a loss of market shares, is large since most consumers are fully informed. Market shares, thus, tend to fluctuate between values near the two extremes. When λ increases, we expect to observe more even distributions of market shares (see Figure 2). However, as λ approaches 3/4, the "attractor" n^* becomes unstable, and two stable fix points (n^{**} and n^{***}) that are located closer to the extremes emerge. Therefore, for large value of λ, we expect to observe skewed distributions of market shares with high frequency again, but this time without the tendency to fluctuate between values near 0 and 1. Instead, market shares become 'sticky' and tend to rest near 0 or near 1 for several consecutive periods. This result can be explained by the 'popularity weighting' property of the word-of-mouth process. If firm 1 has the larger customer base, a large fraction of the consumers in firm 2's customer base find out about firm 1's current offer. The step size upwards (for firm 1) is, thus, larger than downwards for an uneven but not extreme split of the market. This produces the above result. Under strategic interaction (see Section 3.4), firm 1 will, thus, find it easy to "defend" its customer base, while it is difficult for firm 2 to attract new customers if most consumers rely on word-of-mouth communication.

As the above discussion shows, when skewed distributions of market shares are predicted to occur frequently, this leaves open the question whether a firm with a large customer base is

[95] This is because, starting from some $n_1 < n^*$, the first "jump" to the left in the market share space (of firm 1) goes to a point that lies to the left of $1-n^*$. From there, n^{***} is approached. If the starting point is $n_1 = n^*$, the sequence of market shares remains at the unstable fix point forever.

likely to maintain the dominant position[96] in the market for many consecutive periods, or whether market shares tend to fluctuate between values near the two extremes. In the following, a simple measure is introduced that helps to characterize the 'persistence' of a dominant position.

Let k be the minimum number of consecutive market share losses of firm 1, that, starting from $n_1 \equiv 1$, are sufficient to destroy firm 1's dominant position. Only if k is large *and* the information process favors skewed distributions of market shares, then a firm with a dominant position is likely to maintain this position for many consecutive periods under the stochastic benchmark process.

Proposition 1.2: Given the information process specified in (14), for $\rho = 0$ and $n_1 = 1$, the minimum number of consecutive market share losses k, such that firm 1's market share falls below 1/2, is given by the smallest integer greater than:

$$\tilde{k}(\lambda) \equiv \ln\left(1 - \frac{\ln 2}{\ln \lambda}\right) / \ln 2 \tag{38}$$

Proof:

For $\rho = 0$, $l(n)$ (in (14)) simplifies to: $l(n) = \lambda n^2$. Therefore, applying the function $l(.)$ k times to $n_1 = 1$, we obtain for firm 1's market share: $l^k(1) = \lambda^{2^k - 1}$. For $k = \tilde{k}$, this must be equal to 1/2. Solving for $\tilde{k}$, we obtain (38). □

For example, $\tilde{k}(\lambda = 0.79) = 2$, and $\tilde{k}(0.91) = 3$. For values of λ between 0.79 and 0.91, two consecutive market share losses are, thus, sufficient to destroy firm 1's dominant position in the market (starting from $n_1 = 1$).[97] For values of λ greater than, say, 0.95, market shares generated under the stochastic benchmark will rarely cross the 1/2 – 1/2 line (see Section 3.3.2, below).

<u>Case 2</u>:

Using $\lambda = 0$, we obtain for $g(.)$ using (14) and (32):

[96] To have a "dominant position" means that the size of the respective firm's customer base is greater than 1/2.

[97] Under the stochastic benchmark process with $\Pr[p_{1,t} < p_{2,t}] = 1/2$, the probability to observe k consecutive market share losses starting from the current period equals $1/2^k$.

$$g(n) = 1 - \rho\left(1 - \rho n\right) \tag{39}$$

Using (39), one obtains the following solution to (34):

$$n^* = 1/(1+\rho) \tag{40}$$

Obviously, $n^* \in [0,1]$ holds for the entire interval $\rho \in [0,1]$.

Proposition 2.1: Given the information process specified in (14), for $\lambda = 0$ and $\rho < 1$, the fix point n^* is always stable.

Proof: By (39), $\left|dg(n)/dn\right| = \rho^2 < 1$, which holds since $\rho < 1$ by assumption. □

According to (40) and Proposition 2.1, if $\lambda = 0$ and ρ is small, we expect the stochastic benchmark process to fluctuate between values near the extremes. When ρ is close to 1, we expect market shares near 1/2 with high frequency. Market shares are, then, very "sticky" because the step sizes (upwards and downwards) are small. However, in contrast to the case $\rho = 0$ and λ near 1, the market split tends to be an even one. This result can be explained as follows. Since a fixed fraction of consumers in firms' customer bases are searchers, the mass of consumers in firm 1's customer base who find out about firm 2's offer increases in the size of firm 1's customer base. As a consequence, step sizes "downwards" (towards the middle of the market share range) are larger than "upwards" if $n > 1/2$. This explains the tendency to produce even distributions of market shares. Note, that the same effect occurs in the case $\rho = 0$ and $\lambda > 0$. However, as λ gets large, the popularity weighting property of the word-of-mouth process becomes dominant and neutralizes the effect that tends to produce even splits of the market.

Proposition 2.2: Given the information process specified in (14), for $\lambda = 0$, and starting from $n_1 = 1$, the minimum number of consecutive market share losses k, such that firm 1's market share falls below 1/2, is given by the smallest integer greater than:

$$\tilde{k}(\rho) \equiv -\frac{\ln 2}{\ln \rho} \tag{41}$$

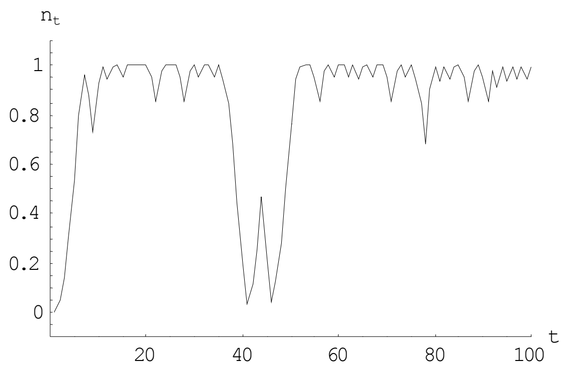
Proof: For $\lambda = 0$, $l(n)$ (in (14)) simplifies to: $l(n) = \rho n$. Applying $l(.)$ k times to $n_1 = 1$, we obtain: $l^k(1) = \rho^k$. For $k = \tilde{k}$, this equals 1/2. Solving for $\tilde{k}$, we obtain (41). □

For example, $\tilde{k}(\rho = 0.71) = 2$, and $\tilde{k}(0.79) = 3$. Note, that in contrast to the case $\rho = 0$ and λ near 1 (see above), a large value of ρ, and, thus, of k does not imply that market shares near the extremes are to be expected with high frequency, because the "attractor" n^* is close to 1/2 if ρ is large. However, it implies that, given that a firm has a large or a small market share at the beginning, an even split of the market is obtained only gradually. An entrant with a customer base initially equal to zero can, thus, increase its market share only gradually (see Section 3.4.2).

3.3.2. The stochastic benchmark

We are now equipped with a number of qualitative predictions about the market share dynamics under the stochastic benchmark. In the following, these predictions are confirmed by means of simulation. The reason why analytical results on the patterns of market share evolution are difficult to obtain even for the simple stochastic benchmark case will become clear below, when the shape of the invariant distribution of market shares is discussed.

The following Figures show simulations of dynamic market share patterns under the stochastic benchmark for different parameter values ρ and λ, and approximations of the invariant distribution of market shares for each set of parameter values. For every parameter set, a different characteristic pattern of market share evolution is obtained. The invariant distributions are also computed by simulation, but with a much higher number of periods.[98] Note, that the invariant distribution shows the frequency of each possible state, but it does not reveal the patterns of *transition* between the states, e.g., whether a high market share of one firm is often maintained for many consecutive periods, or whether market shares tend to fluctuate between the extremes.



[98] usually between 10^5 and 10^6 periods; for the market share grid of the approximated invariant distribution I used a uniform step size of 0.01

Figure 3a: Simulated evolution of market shares, stoch. benchmark; λ = 0.95, ρ = 0, α = 1

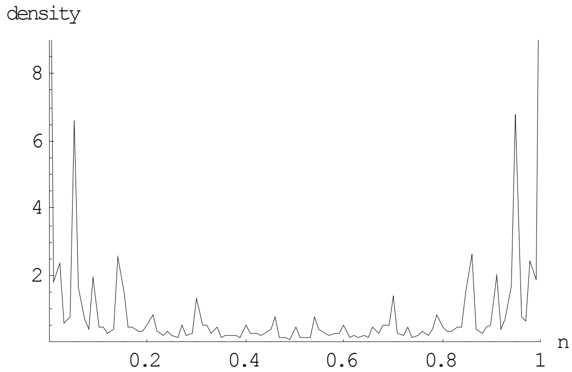


Figure 3b: distribution of market shares, stoch. benchmark; λ = 0.95, ρ = 0, α = 1

Figure 3a shows a simulated evolution of market shares for $\lambda = 0.95$, $\rho = 0$, and $\alpha = 1$. Using (37) and (14), we obtain the fix point $n^{**} = 0.997$, and $l(n^{**}) = 0.944$. By (38), $k = 4$ consecutive market share losses are necessary to destroy firm 1's dominant position if $n_1 = 1$. Market shares near the extremes are, thus, predicted to occur with high frequency, and a firm with a dominant position is likely to maintain this position for many consecutive periods. The predictions are confirmed by Figure 3a.

Figure 3b shows the invariant distribution for the same parameter values, and illustrates that it has a highly irregular shape. The highest peaks at the extremes in Figure 3b correspond to the fix point computed above. The irregular shape of this distribution gives an idea why quantitative results are hard to obtain analytically.

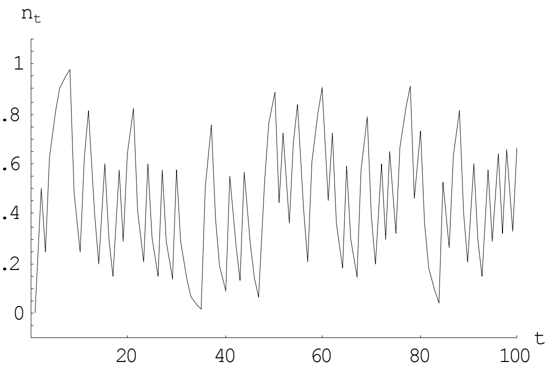


Figure 4a: Simulated evolution of market shares, stoch. benchmark; λ = 0, ρ = 0.5, α = 1

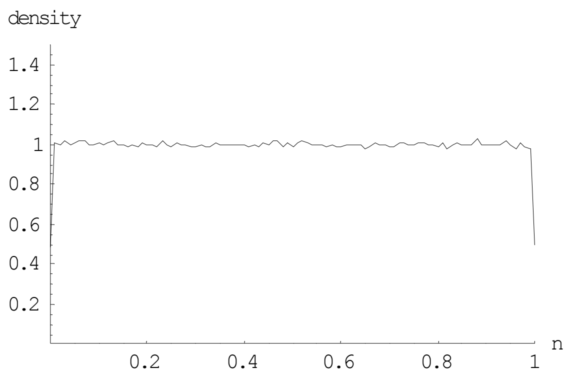


Figure 4b: Invariant distribution of market shares, stoch benchmark; λ = 0, ρ 0.5, α = 1

Figure 4a is a simulation for $\lambda = 0$, $\rho = 0.5$, and $\alpha = 1$. Using (40), we obtain the fix point $n^* = 2/3$, and $l(n^*) = 1/3$. According to (41), a single loss of market share is always sufficient to destroy a firm's dominant position. Therefore, if a firm obtains a dominant position, it is unlikely to maintain this position for several consecutive periods. Figure 4b shows the approximated invariant distribution for $\lambda = 0$ and $\rho = 0.5$. Interestingly, for these parameter values, the invariant distribution is uniform.[99] All market shares in the open interval (0,1) occur with the same frequency. However, if ρ differs only slightly from 1/2, highly irregular invariant distributions are obtained again (not shown).

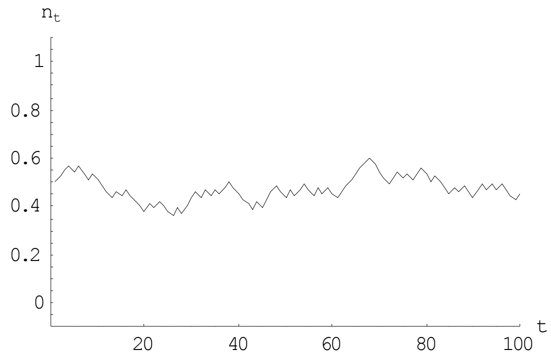


Figure 5a: Simulated evolution of market shares, stoch. benchmark; λ = 0, ρ 0.95, α = 1

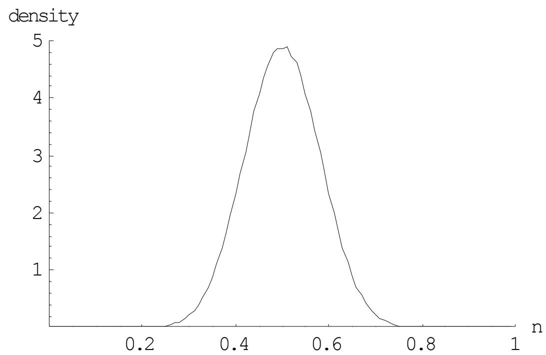


Figure 5b: Invariant distribution of market shares, stoch. benchmark; λ = 0, ρ 0.95, α = 1

Figure 5a shows a simulation for $\lambda = 0$, $\rho = 0.9$, and $\alpha = 1$. According to the results of Section 3.3.1, market shares near $n^* = 0.53$ and $l(n^*) = 0.47$ should occur with high frequency, and this is confirmed. Figure 5b shows the approximated invariant distribution. The tendency towards even splits of the market is confirmed. Unlike for smaller values of ρ,

[99] The small bumps are due to numerical imprecision. Unlike the ones in Figure 3b, they become smaller as the number of periods is raised.

a regular shape is now obtained. The comparison of Figures 3a and 5a illustrates that the presence of word-of-mouth consumers and consumers of the ignorant type can lead to markedly different market share dynamics. When there is a large fraction of word-of-mouth consumers in the population, market shares tend to be skewed, and a firm with a large customer base tends to maintain the dominant position in the market for many consecutive periods. When most consumers are of the ignorant type, market shares tend to fluctuate around 1/2.

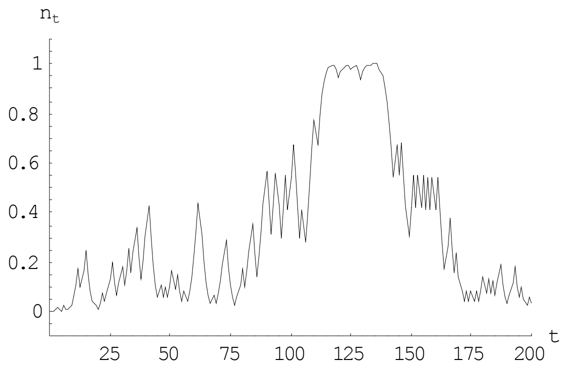


Figure 6a: Simulated evolution of market shares, stoch. benchm.; λ = 0.49, ρ = 0.49, α = 1

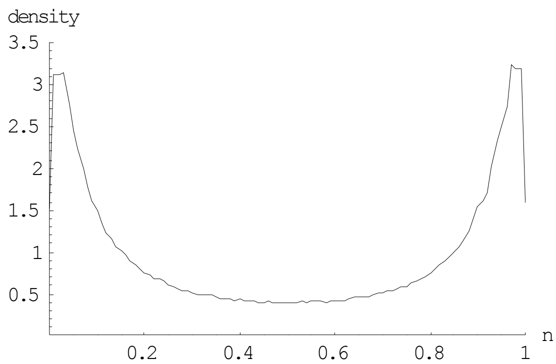


Figure 6b: Invariant distribution of market shares, stoch. benchm.; λ = 0.49, ρ = 0.49, α = 1

Figure 6a shows a simulation with word-of-mouth consumers and ignorant types. Note, that the sum of λ and ρ must be strictly below one, for otherwise, a firm that obtains a market share of 1 will maintain it forever.[100] The simulated process shows no clear tendency towards even or skewed distributions of market shares, but the approximated invariant distribution in Figure 6b reveals that skewed distributions occur with higher frequency than even ones.

If the frequency of purchases α is below one, the dynamic evolution of the state variable is not identical to the evolution of the realized market shares. The latter is more volatile than the former if $\alpha < 1$. This is illustrated in Figure 7a that shows a simulation for the stochastic

[100] This is because nobody finds out about the offer of a firm with no customer base if there are no searchers.

benchmark, for $\lambda = 0.95$, $\rho = 0$, and $\alpha = 0.2$. Figure 7b shows the approximated invariant distribution of realized market shares for these parameter values.

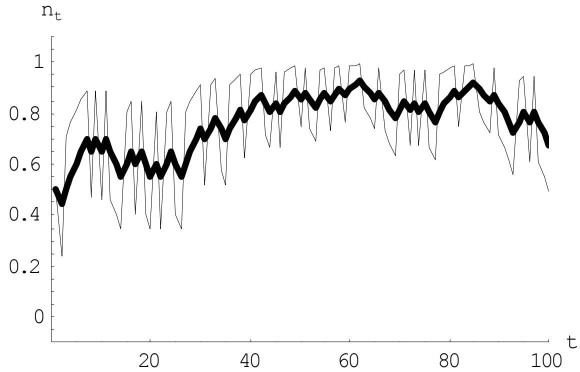


Figure 7a: Simulated evolution of state and market shares, stoch. benchmark; α = 0.95, ρ = 0, α = 0.2 (thick curve: evolution of the state, thin: realized market shares)

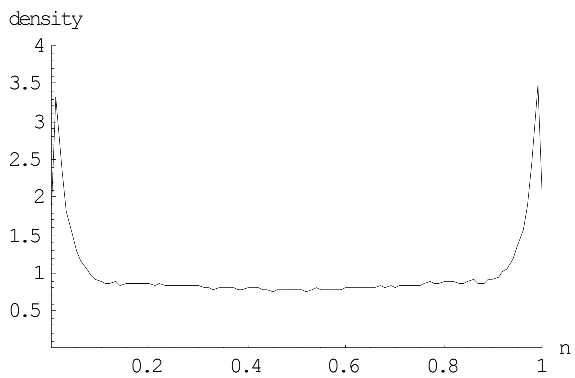


Figure 7b: Invariant distribution of market shares, stoch. b.m.; α = 0.95, ρ = 0, α = 0.2

The thin curve in Figure 7a, that represents firm 1's realized market shares, oscillates around the thick curve that shows the evolution of the state n. The comparison of Figures 7a and 3a, that were obtained for the same parameters except α, shows that the state and the realized market shares tend to be less extreme for $\alpha = 0.2$ than for $\alpha = 1$, but the volatility of realized market shares is higher for $\alpha = 0.2$. This is because for $\alpha = 1$, market shares are near 0 or 1 most of the time, and step sizes are smaller near the extremes. The comparison of Figures 7b and 3b illustrates that the invariant distribution becomes regular shaped when the failure rate α is reduced (this holds for large values of λ). As a consequence, if the iteration procedure introduced in Section 3.2 fails to converge for large values of δ and λ when $\alpha = 1$, it may converge when α is reduced.

3.4. MARKET SHARE DYNAMICS UNDER FIRMS' STRATEGIC INTERACTION

In Section 3.2, it was shown that the equilibrium of the game is in mixed strategies and, thus, yields price dispersion that generates non-trivial market share dynamics. In this Section, these market share dynamics are analyzed. There are two cases that are analyzed separately: 1. the repeated static game / myopic firms (Section 3.5.1), and 2. the full-fledged dynamic game (Section 3.5.2). While in the latter, firms' incentives to invest (or disinvest, depending on the shape of the value function) in future market shares play an important role, they are excluded in the repeated static game. However, the forces that drive the dynamic evolution of market shares in the repeated static game are also present when future profits are important. Market share dynamics in the dynamic game with $\delta > 0$ should, thus, be discussed in comparison with the repeated static game which, in turn, is compared to the stochastic benchmark case without strategic interaction.

3.4.1. Market share dynamics in the repeated static game

The static game can be solved analytically.[101] The solution is given by the iteration formula (29) (for $\delta = 0$), together with (14), (25), and (14). As in Section 3.3.2, the dynamic evolution of market shares is analyzed by means of simulation. By convention, $\alpha = 1$ holds unless stated otherwise (each consumer makes a purchase in each period).

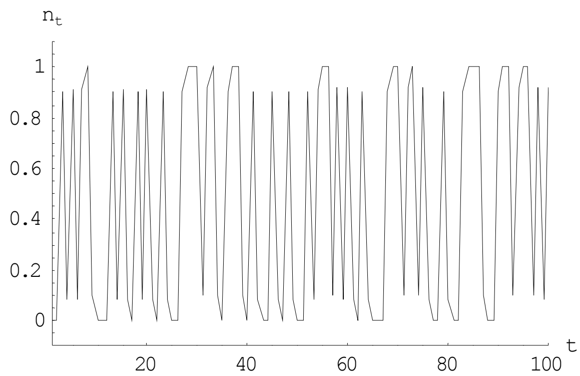
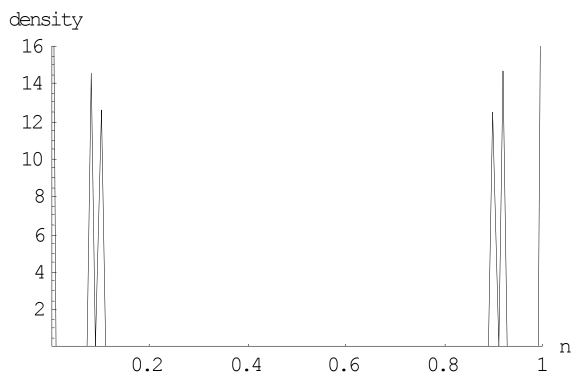


Figure 8a: Dynamic evolution of market shares; $\lambda = 0.1$, $\rho = 0$, $\delta = 0$, $\alpha = 1$



[101] See chapter 2, where the shape of the past sales – current profit relation is analyzed in the static case (market share dynamics are not discussed). In the present chapter, the focus is on market share dynamics.

Figure 8b: Invariant distribution of market shares; λ = 0.1, ρ = 0, δ = 0, α = 1

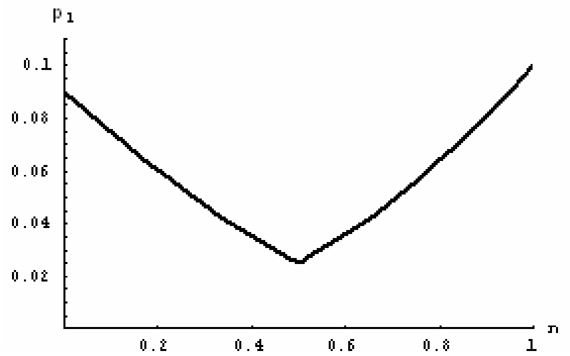


Figure 8c: Value function V(n); λ = 0.1, ρ = 0, δ = 0, α = 1

Figure 8a shows a simulation of the dynamic evolution of market shares for $\lambda = 0.1$ and $\rho = 0$. It illustrates that for λ and ρ near zero, market shares are very volatile and oscillate between values of n near zero and near one. This case is interesting because it is only a small step away from the standard Bertrand model with fully informed consumers. The evolution of market shares in Figure 8a is far from being a plausible prediction for most markets in the real world, where market shares are less volatile, and firms can often maintain a dominant position in a market over a long period of time. This indicates that consumers are less well informed in the real world, or other mechanisms (such as superior production technologies or product innovations) must be at work that help firms sustain a dominant market position.

Figure 8b shows the approximated invariant distribution of market shares, and reveals that there are roughly four different splits of the market that can occur starting from period 2 onwards. One of them ($n \cong 0.9$) is obtained if firm 1 gains market shares when the size of its customer base is near zero, and the other if it gains market shares once more afterwards (it, then, serves almost the entire market: $n \cong 1$). The other two splits follow from symmetry.

These findings can be compared with the stochastic benchmark case. It turns out that for $\lambda + \rho$ near zero, the stochastic benchmark yields very similar dynamics as the repeated static game (not shown). This is related to the fact that the probability of gaining or losing market shares is always near 1/2 (irrespective of n).[102]

It is interesting to note, that for small values of λ and ρ, the value function (that for $\delta = 0$ simply reflects firm 1's current expected profit as a function of n) is V-shaped (see Figure 8c). This result is related to the fact that price competition is particularly intense for even ex-

[102] In the repeated static game, the probability to maintain a dominant market position is somewhat lower than in the stochastic benchmark because the firm with the smaller customer base prices more aggressively, so the firm with the dominant market position loses market shares with a probability (slightly) greater than 1/2.

ante splits of the market.[103] It implies that firms prefer to enter the market either with a small or with a large customer base. Whether this gives firms an incentive to invest or disinvest in future market shares when $\delta > 0$ is discussed in Section 3.4.2, below.

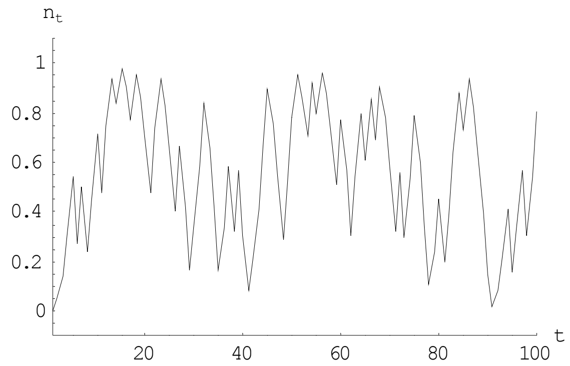


Figure 9a: Dynamic evolution of market shares; $\lambda = 0.95$, $\rho = 0$, $\delta = 0$, $\alpha = 1$

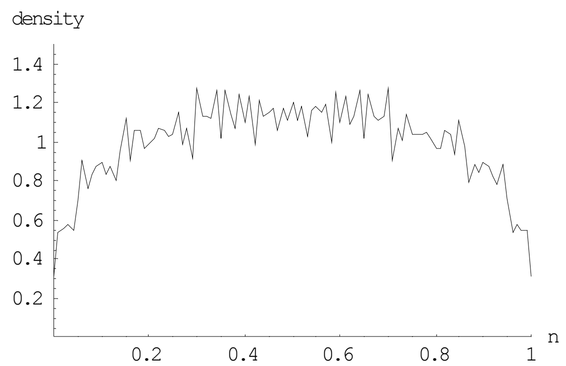


Figure 9b: Invariant distribution of market shares; $\lambda = 0.95$, $\rho = 0$, $\delta = 0$, $\alpha = 1$

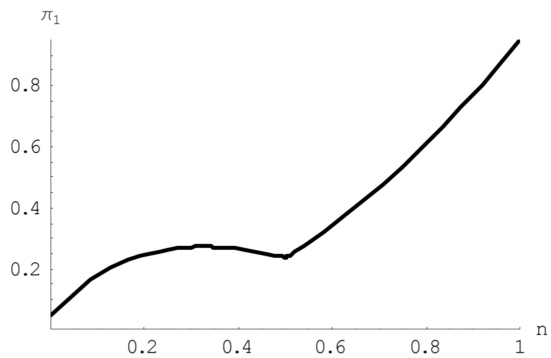


Figure 9c: Value function V(n); $\lambda = 0.95$, $\rho = 0$, $\delta = 0$, $\alpha = 1$

Figure 9a illustrates the characteristic evolution of market shares for $\lambda = 0.95$ and $\rho = 0$. For these parameters, the stochastic benchmark showed a strong tendency towards skewed market

[103] See chapter 2.

shares and a high degree of persistence near the extremes[104] (see Figure 3a), reflecting the popularity weighting property of the word-of-mouth process. Under strategic pricing, market shares are on average less skewed, and there is less persistence at the extremes. This is related to the fact that a firm with a small customer base prices aggressively in order to gain market shares. A firm with a large customer base is, thus, likely to lose market shares, which yields a tendency towards even splits of the market. This effect partly neutralizes the tendency inherent in the word-of-mouth process to produce market shares near the extremes.

Figure 9b shows the invariant distribution for $\lambda = 0.95$ and $\rho = 0$. It is irregular, but confirms the tendency towards more even splits of the market. Figure 9c illustrates that firm 1's value function has a local maximum between $n = 0$ and $n = 1/2$ for large values of λ. This is related to the fact that, starting from an even ex-ante split of the market (n near 1/2), firm 1 benefits from a small reduction in the size of its customer base as this softens price competition. A larger reduction is not beneficial because a firm with a very small customer base is in an unfavorable position in a market where most consumers rely on word-of-mouth as almost nobody will find out about this firm's offer.

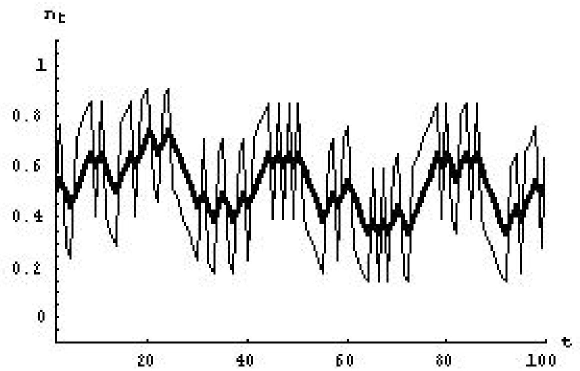


Figure 10a: Evolution of state and market shares; $\lambda = 0.95$, $\rho = 0$, $\alpha = 0.2$, $\delta = 0$

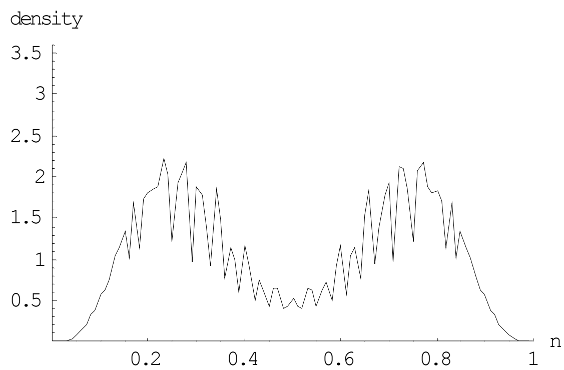


Figure 10b: Invariant distribution of market shares; = 0.95, $\rho = 0$, $\alpha = 0.2$, $\delta = 0$

[104] This means that a firm with a dominant position in the market is likely to maintain this position for many consecutive periods. Remember, that for small values of λ and ρ, market shares are skewed, too, but fluctuate between the extremes.

Figure 10a shows a simulation of the evolution of market shares for $\lambda = 0.95$, $\rho = 0$, and $\alpha = 0.2$. The invariant distribution in Figure 10b reveals a bimodal shape. This can be explained as follows. The state variable n is not very volatile and tends to be near 1/2. Firm 1's realized market shares to fluctuate around the state variable, and, thus, around 1/2. The average step size is around 0.5, so the highest peaks in the invariant distribution have roughly this distance.

3.4.2. Market share evolution in the full-fledged dynamic game (δ > 0)

The evolution of market shares for $\delta > 0$ is analyzed using simulations based on the approximated value function $\tilde{V}(n)$. The value function is approximated using the iteration procedure or the method of discretizing the state space (see Section 3.2).[105] Note, that the dynamic evolution of market shares and the shape of the value function are closely linked and can not be understood in isolation. The link is as follows. The shape of the value function implies the shape of the price distribution functions via (14) for every possible state n. The price distribution functions generate (for the given information process) the stochastic process of market share evolution. The value function itself, however, depends on the properties of this stochastic process.

The first result is that, for values of λ and ρ near zero, market share dynamics for $\delta > 0$ are very similar to those for $\delta = 0$ (see Figure 8a). The reason for this is the following. If $\lambda + \rho$ is near zero, most consumers are fully informed. Therefore, irrespective of the size distribution of firms' customer bases, the firm that currently offers the lower price serves most of current market demand. Firms, thus, have little incentive to invest or disinvest in future market shares. The shape of the value function and market share dynamics are, thus, similar to the ones for $\delta = 0$.

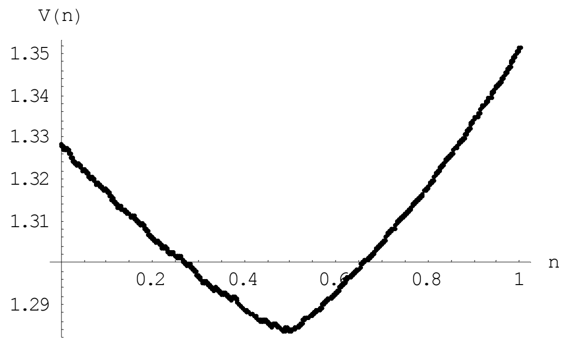


Figure 11: Approx. value function $\tilde{V}$(n); λ = 0.1, ρ = 0, δ = 0.95

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[105] The procedure with the discretized state space is computationally faster when the iteration process takes many rounds to converge. Both methods yield (up to some numerical imprecision) identical results.

Figure 11 shows the approximated value function $\tilde{V}(n)$ for $\lambda = 0.1$, $\rho = 0$, and $\delta = 0.95$. It illustrates that the V-shape is preserved when δ is raised (see Figure 8c). However, the value function is shifted upwards because profits accumulate over several periods. Since competition in future periods is almost unaffected by the current market outcome, future expected profits are essentially a constant that is added to current expected profit.

In sum, when most consumers are fully informed, market shares are very volatile and tend to oscillate between values near 0 and near 1, irrespective of the discount factor for future profits. Furthermore, the value function for this case is typically V-shaped, because current price competition is particularly intense for an even *ex-ante* split of the market.[106]

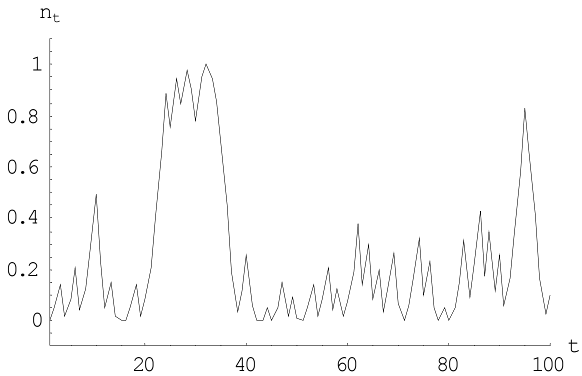

Figure 12a: Dynamic evolution of market shares; λ = 0.95, ρ = 0, δ = 0.5

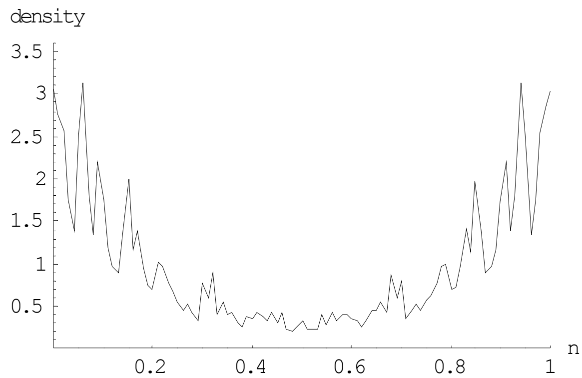

Figure 12b: Invariant distribution of market shares; λ = 0.95, ρ = 0, δ = 0.5

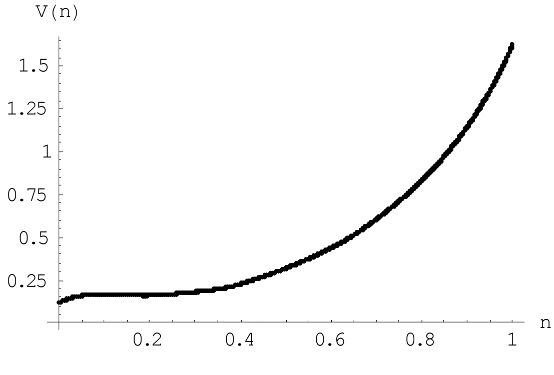

Figure 12c: Approx. value function $\tilde{V}(n)$; λ = 0.95, ρ = 0, δ = 0.5

[106] See chapter 2.

Figure 12a shows a simulation of the dynamic evolution of market shares for $\lambda = 0.95$, $\rho = 0$, and $\delta = 0.5$. As the comparison with Figure 9a shows, for larger values of the discount factor δ, market shares tend to be more skewed, and a firm that has obtained a dominant position in the market is more likely to maintain this position for several consecutive periods. The evolution of market shares in Figure 12a is more similar to the stochastic benchmark case (see Figure 3a) than to the one for $\delta = 0$ (Figure 9a).

The results can be explained as follows. When future profits are important, a firm with a large customer base (say, firm 1) has an incentive to *defend* its dominant position in the market. This leads to more intense price competition near the extremes, which partly neutralizes the tendency towards even market splits observed for $\delta = 0$. This effect is particularly strong when n is not equal to 1, but lies somewhere between 0.5 and 1. This can be confirmed by analyzing the location of the price distribution functions for different values of n. It turns out that for $n = 1$, firm 1's price distribution function first order stochastically dominates firm 2's, so firm 1 is likely to lose market shares. For lower values of n (between 1/2 and 1), the situation is reversed, and firm 1 is likely to gain market shares. This explains why in Figure 12a, market shares tend to oscillate between 0.7 and 1 (or, by symmetry, between 0 and 0.3), and rarely remain at the extremes for several periods.

Figure 12b shows the approximated invariant distribution for the same parameters. The comparison with Figure 9b confirms the observation that market shares tend to be more skewed for larger values of δ. The invariant distribution is highly irregular, which indicates that for larger values of the discount factor δ, and λ near 1, the dynamic program may not have a solution (feedback effects become important, and a slight change in the size of a firm's customer base may imply a substantial change in the firm's value). It turns out that the iteration procedure fails to converge if δ is increased beyond, say, 0.6. However, if the frequency of purchases α is reduced, the irregularities disappear from the invariant distribution, and the model can be solved also for large values of δ (see below).

Figure 12c shows the approximated value function $\tilde{V}(n)$ for $\lambda = 0.95$, $\rho = 0$, and $\delta = 0.5$. The local maximum that was obtained for $\delta = 0$ (see Figure 9c) is no longer obtained. The relation between a firm's value and the size of its customer base is now monotonously increasing. Interestingly, for values of n around 1/3, the expected profit in the dynamic case (Figure 12c) is actually *lower* than for $\delta = 0$ (see Figure 9c), despite the fact that profits are accumulated over several periods. This can be explained as follows. For n near 1/3, the firm with the smaller customer base (firm 2) is willing to fight for the dominant position in the market because it has a realistic chance to achieve this, while firm 1 tries to defend its dominant position. This leads to intense price competition. At lower values of n, firm 2 can not obtain the dominant position in the market within a small number of periods (due to the popularity weighting property of the word-of-mouth process). It, thus, constitutes a 'weak competitor' to firm 1, which softens price competition.

In sum: when most consumers rely on word-of-mouth, and the discount factor δ is greater than zero, the typical evolution of market shares is as follows. One of the firms obtains a market share near 1. This is followed by an exploitation of its market power over the uninformed repeat purchasers, which leads to a loss of market shares. This is followed by an effort to maintain the dominant position in the market, that usually leads to a gain of market shares again. Market shares, thus, tend to fluctuate between values above 1/2 and near 1 (or, by symmetry, below 1/2 and near 0), and are on average more skewed than for $\delta = 0$. The value function is monotone.

Markets with finitely durable goods / Entry:

In the following, I want to analyze how a reduction in the frequency of purchases α affects the dynamic evolution of market shares when $\delta > 0$. The idea that not every consumer makes a new purchase in each period adds an important aspect of real-world markets to the model. It helps to explain why firms can enter only slowly into existing markets, and why firms are so concerned about market share targets in general.

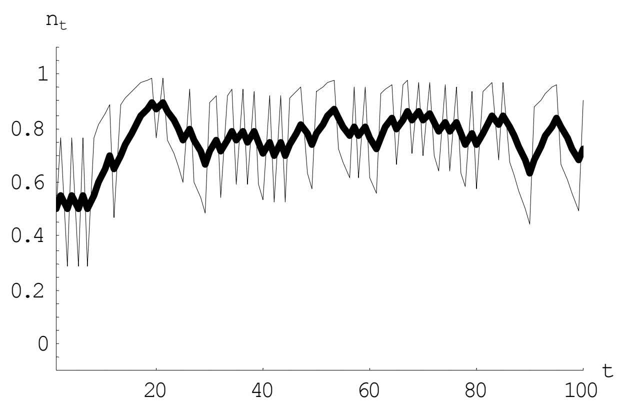

Figure 13a: Evolution of state and market shares; $\lambda = 0.95$, $\rho = 0$, $\alpha = 0.2$, $\delta = 0.7$

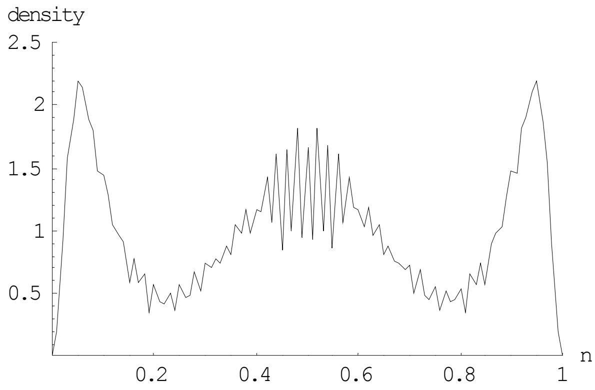

Figure 13b: Invariant distribution of market shares; λ = 0.95, ρ = 0, α = 0.2, δ = 0.7

Figure 13a shows the simulated evolution of the state and realized market shares for $\lambda = 0.95$, $\rho = 0$, $\alpha = 0.2$, and $\delta = 0.7$. It is similar to the one under the stochastic benchmark shown in Figure 7a. Firm 1's realized market shares oscillate around the state, and the state moves only slowly. The invariant distribution shown in Figure 13b reveals an interesting result: market shares near the extremes and even splits of the market occur more frequently than market shares near 0.8 (or 0.2). This is related to the fact that the invariant distribution of the state has two peaks located at about 0.2 and 0.8 (not shown). Since firm 1's realized market shares tend to oscillate around the state, we observe the two areas with low frequencies around 0.2 and 0.8 in Figure 13b. The intuitive explanation for this result is similar as for $\alpha = 1$ (see above). Given that firm 1's total customer base size, including all consumers who purchased from firm 1 when they last made a purchase, is large, firm 1 tends to exploit the uninformed consumers in its customer base, and, thus, tends to lose market shares. For lower values of n (between 1/2 and 1), firm 1 tries to defend the dominant position in the market. As a consequence, the state tends to remain near 0.8 (or 0.2) for many consecutive periods, and firm 1's realized market shares oscillate around the state.

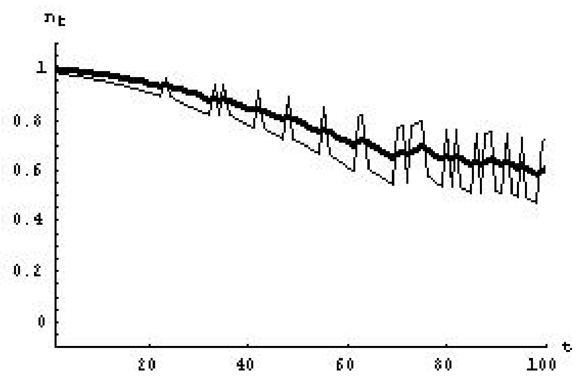

Figure 14: Evolution of state and market shares; λ = 0.49, ρ = 0.49, α = 0,1, δ = 0.5

Figure 14 illustrates the effects of incomplete consumer information ($\lambda + \rho$ near 1) and a low frequency of purchases ($\alpha = 0.1$) on the dynamics of entry into the market. The entrant (firm 2) starts with a customer base of size 0, and gradually captures a greater share of the market by undercutting the incumbent's price. The state gradually approaches values near 1/2, where most of the probability mass is located in the state's invariant distribution for the given parameter values (not shown). As n decreases, market shares start to fluctuate around the state. Figure 14 nicely illustrates that entry into a market with incomplete consumer information (that induces customer loyalty towards incumbent suppliers) and a low frequency of purchases is time consuming and requires the entrant to price more aggressively than the incumbent over many periods.

3.5. CONCLUSION

Although repeat purchasing appears to be the rule rather than the exception in most markets for consumption goods, in search models, it is usually assumed that knowledge from previous consumption experiences plays no role. A static modeling framework, thus, seems appropriate. Within this framework, a typical feature of search games are mixed pricing equilibria that generate price dispersion. Firms' market shares are, thus, not constant over time. But in the absence of an intertemporal link, these dynamics are rather trivial, so authors paid little attention to them. The empirical literature, however, emphasizes that market shares tend to have a high persistence over time, that is, skewed or even splits of the market are usually preserved for extended periods of time.[107] This suggests that there must be an intertemporal link that is not captured by the simple repetition of a static search game.

In the present chapter, an intertemporal link is introduced by assuming that consumers are repeat purchasers who possess superior information about the product they purchased previously. Combined with the price dispersion generated by the firms' use of mixed pricing strategies, this leads to non-trivial market share dynamics. In brief, when many consumers search actively, market shares are very volatile, and firms have little incentive to invest in future market shares. The dynamics under strategic profit maximizing behavior, thus, resemble those generated by a simple stochastic process that assigns equal probabilities of gaining or losing market shares to both firms in each period. When many consumers rely on word-of-mouth communication as a source of information, market shares tend to be skewed, and a firm with a large customer base tends to maintain the dominant position in the market for many consecutive periods. This is especially true if the discount factor for future profits is

[107] See, e.g., Pakes (1987).

sufficiently large, which gives a firm with a dominant position in the market an incentive to defend its customer base against an aggressive competitor with a smaller customer base. If consumers do not make a purchase in each period, market shares tend to be more sticky. This helps to explain why entry into a market often takes so much time (in standard models such as the Cournot model, an entrant immediately obtains its long run market share), and why firms are so concerned about market share targets in general.

3.6. APPENDIX

Proof of Lemma 1:

Proof by contradiction. Suppose there is a Markov perfect equilibrium that assigns the pure strategies p_1 and p_2 to the state $n \in [0,1]$. It must be shown that, for any p_1 and p_2 ($p_1, p_2 \leq 1$), at least one of the firms has an incentive to deviate. If $p_1 = p_2 \equiv p$ and $p > 0$, either firm would benefit from marginally undercutting the common price level p as this leads to a discontinuous rise in current demand that, for a sufficiently low δ, more than compensates for potential future losses resulting from the increase in the size of the firm's customer base. The case $p_1 = p_2 \equiv p$ and $p \leq 0$ can not be an equilibrium because, if it were an equilibrium, the state n would remain constant, so firms choose $p_1 = p_2 \equiv p$ in all periods and total discounted profits are non-positive, while deviating to the monopoly price 1 yields a positive profit to a firm with a strictly positive customer base (and there is at least one such firm). If $p_1 \neq p_2$ and $p_1, p_2 < 1$, the high-priced firm would benefit from deviating to the monopoly price because current demand and, thus, the state next period are not affected. There can be no equilibrium where $p_i = 1$ and $p_{j \neq i} < 1$, as the low-priced firm would benefit from deviating to a higher price, or the high-priced firm from marginally undercutting the lower price (or both).

Proof of Lemma 2:

Let S_1 and S_2 be the supports of, respectively, $F(n,p)$ and $F(1-n,p)$. Suppose $S_1 \neq S_2$ for a given value of n. Therefore, there is a price $\tilde{p} < 1$, with $\tilde{p} \in S_i$ but $\tilde{p} \notin S_j$ ($i, j \in \{1,2\}$, $j \neq i$). This can not be an equilibrium because, since $\overline{S_j}$ is open, there exists a price $p > \tilde{p}$ with $p \notin S_j$ that yields the same expected demand to firm i in the current period as $\tilde{p}$, but a higher profit if δ is sufficiently small. This holds if p is above the maximum of S_j (if the maximum is below 1), below the minimum of S_j, or within some intermediate range that is not part of S_j when S_j is not convex. Therefore, $S_1 = S_2 \equiv S$. The maximum of S must be the monopoly price 1, because otherwise, each firm would benefit from deviating to the monopoly price. This yields the same demand as the maximum of S but a higher profit. The minimum of the support, $\underline{p}$, is smaller than 1 since (by Lemma 1), there is no pure strategy equilibrium. Furthermore, S is convex. Suppose to the contrary that there is an intermediate range that is not part of S. Firm i's expected demand would be constant over this range, but expected profit would be increasing. Therefore, expected profit would be higher in the upper interval of S. This can not be an equilibrium. □

Proof of Lemma 3:

Suppose both firms attach positive probability mass to some identical price level p in $S = [\underline{p}, 1]$ when the current state is n. Each firm would, then, benefit from shifting its mass point to a price level marginally below p because this leads to a discontinuous rise in current expected demand, which always benefits the firm if δ is sufficiently small. Strategies containing a single mass point at some price p in the interval $(\underline{p}, 1)$ can not be an equilibrium either since the competitor's expected demand would fall discontinuously at p. There can be no equilibrium where the distribution function of one firm contains a mass point at $p = \underline{p} > 0$ as the competitor's current expected profit would be larger at prices marginally below $\underline{p}$ than at prices above $\underline{p}$. This can not be an equilibrium since prices below $\underline{p}$ are not part of the support. There can be no equilibrium where the distribution function of one firm contains a mass point at $p = \underline{p} \leq 0$ as the firm would benefit from shifting the mass point to some higher price level.

Chapter 4. On the Regulation of Vertically Differentiated Markets with more than two Firms

4.1. Introduction

It has long been recognized that in markets where firms choose not only prices but also the quality level of their products, market outcomes can lead to inefficiencies in the provision of quality. This view is supported by the widespread usage of policy measures, in particular minimum quality standards, that aim at improving the overall provision of quality in markets. There is, however, no general agreement among theorists about the reasons behind the inefficient provision of quality under market conditions. The literature offers two types of explanation that are surprisingly divergent.

One strand of literature (see e.g. Leland, 1979; Shapiro, 1983; Albano and Lizzeri, 2001) refers to incomplete consumer information as the primary source of inefficiencies in vertically differentiated markets. If consumers can not observe all relevant product characteristics before purchase (experience goods), or possibly not even after purchase (credence goods), then firms may be in a position to exploit them by offering poor quality at excessive prices. In the worst case, the market can collapse (see Akerlof's market for Lemons, 1970). There are, of course, remedies to this problem. E.g., when consumers repurchase a product frequently, then firms can build up a reputation for being a high quality provider. Shapiro (1983) shows in a competitive modeling framework, that high quality providers charge prices above marginal costs. The prices are just high enough to deter them from cheating (by cutting quality). The rents, however, only compensate the firms for initial losses when building up their reputation. Another possibility is that firms let external experts test / certify their products to overcome the information asymmetry (see Albano and Lizzeri, 2001). However, this may not always be the most efficient way to eliminate the information asymmetry between consumers and firms. Furthermore, even if consumers are provided with very detailed product information, they may sometimes find it difficult to correctly interpret this information.[108] Minimum quality standards, thus, often specify upper limits to chemicals or other inputs.

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[108] E.g., how can a consumer tell what is an acceptable amount of certain chemicals or ingredients in food, or of radiation from a microwave oven etc.?

A second stand of literature abstracts from information asymmetries, and explains inefficiencies as the outcome of firms' strategic interaction or monopolistic exploitation. In particular, it has been shown that duopolistic firms over-differentiate in the quality space in an attempt to soften price competition (see Cremer and Thisse, 1994). This result should not be confused with the "principle of differentiation" (Tirole, 1988), according to which firms do not choose identical positions in the quality space in order to avoid marginal cost pricing. Whereas the principle of differentiation only says that two firms do not offer identical qualities in equilibrium, the over-differentiation result implies that qualities are *too dispersed* from a welfare perspective. This means that the low-quality provider in a duopoly underprovides quality, whereas the high-quality provider offers a product of an excessively high quality.

This chapter contributes to the second strand of literature, namely to the analysis of firms' strategic interaction in oligopolistic markets with endogenous quality choice and fully informed consumers. However, it also very clearly shows the limits to this approach in explaining inefficiencies in the provision of quality. I extend the analysis of previous works (that were mainly focused on duopolies) to a larger number of firms. The main result of the present chapter is that the over-differentiation result that is obtained for duopolistic markets almost entirely disappears as soon as three or more competitors are in the market. The reason behind this is, that there is a regime change in the nature of competition at the transition from two to three (or more) competitors. Intuitively, in a duopoly, firm 1 (the low-quality provider) has an incentive to differentiate towards lower quality levels in order to soften firm 2's pricing behavior, and vice versa. When consumers are fully informed, the incentives are so strong that firms locate even outside the range of consumers' preferred quality levels[109]. In a triopoly, firm 1 has two competitors. Whereas firm 1 only competes for consumers at the margin with its direct neighbor (firm 2), firm 2 is also engaged in a price competition with firm 3. Therefore, when firm 1 lowers its quality choice in order to soften price competition, this will have a weaker effect than in the duopoly case (firm 2's responsiveness to a quality reduction by firm 1 is lower). As a result, the degree of over-differentiation (as compared to the social planner's case) is much smaller in markets with three or more competitors than in duopolistic markets. A direct consequence of this is that policies that improve the overall provision of quality (and hence welfare) in a duopoly, may not be effective or even harmful in markets with three or more competitors. The adequacy of policy prescriptions, thus, depends crucially

[109] A consumer's *preferred quality* level is defined as the consumer's product choice when all feasible quality levels are offered to the consumer, each of them at its respective marginal cost.

on the number of competitors, which implies that some of the previous results are applicable only to strictly duopolistic markets.

The modeling framework I use resembles the one in Cremer and Thisse (1994) but is slightly more general. The authors analyze the effects of an ad valorem tax upon the provision of quality and prices in a vertically differentiated duopoly. They show that a uniform ad valorem tax is similar to a shift in the firms' marginal cost function and leads to lower quality levels and lower prices. The high quality level, thus, moves closer towards the average consumer's preferred quality, and in conjunction with lower prices, welfare unambiguously increases when a moderate ad valorem tax is introduced.

Constantatos and Sartzetakis (1999) extend Cremer and Thisse's (1994) work to allow for entry into a vertically differentiated market. They use a modeling framework that is similar to the one by Shaked and Sutton (1982 and 1983), and consider situations where the market is a natural oligopoly (in the sense of Shaked and Sutton) before the ad valorem tax is introduced.[110] The tax effectively shifts the marginal cost curve upwards, which implies that the range of preferred quality levels shifts downwards. If the tax rate is sufficiently high, the market may switch from natural monopoly / oligopoly to a market that can support an infinite number of competitors. The authors show that, although for a given number of firms, the tax leads to a reduction in quality levels, quality jumps upwards can occur whenever the tax rate reaches a level that induces a change in the endogenous market structure.

Ronnen (1991) analyzes the effects of a minimum quality standard upon the provision of quality and prices in a vertically differentiated duopoly where firms have to pay a quality dependent fixed cost before competing in prices. The author shows that, when the standard is binding for the low-quality provider, also the high-quality provider increases its quality in an attempt to cushion the adverse effects on price competition. Nevertheless, prices fall, and the consumers' participation in the market increases. As a result, a moderate minimum quality standard increases welfare.

Crampes and Hollander (1995) analyze the effects of a minimum quality standard in a duopoly without quality dependent fixed costs, but where the marginal cost of production

[110] For a vertically differentiated market to be a natural oligopoly (which means that only a limited number of firms enter even when entry costs are zero), one must assume that there exists an upper bound to the set of quality levels that *can* be produced, and that this bound lies strictly below the preferred quality level of the consumer with the lowest valuation for quality / the lowest income. The assumption of an exogenous upper bound to quality appears to be quite restrictive and is relaxed in this chapter. The "natural oligopoly" case defined in Shaked and Sutton (1983) does, thus, not apply, so the market can support an infinite number of firms.

increases in quality. Similarly as Ronnen, the authors find that an appropriately chosen standard can improve welfare if the high-quality provider does not raise quality too sharply in response to the increased low-quality level.

In this chapter, I do not explicitly analyze the welfare effects of minimum quality standards or commodity taxation. Instead, I perform the welfare analysis on a more general level. I investigate what the maximum *scope for welfare improvements* is within a given market structure, independently of the specific type of regulation chosen. To this end, I use extreme parameter values that allow me to compute an upper bound to the percentage welfare increase that can be achieved by regulation, using the social planner's solution as reference point. While in a duopolistic market, the scope for welfare improvements is potentially large, it turns out that there is little scope for welfare improvements in unregulated markets with three or more competitors. Therefore, any type of regulation (although perhaps effective in a duopoly) is likely to be harmful in markets with more than two competitors. This includes policy measures such as minimum quality standards or ad valorem taxes.[111] A laissez-faire policy, thus, seems generally appropriate for vertically differentiated markets with well-informed consumers where three or more firms are already in the market.

Given the above results, the question arises where to look for an optimal policy when the market is a duopoly. Should the regulator try to force qualities closer to the optimum within the given market structure, or is it more efficient to offer a subsidy to entry and let competition do the rest? To give an answer to this question, I first analyze whether a subsidy to entry *can* be welfare improving. The answer is "yes, if entry costs are not too high". I, then, compare welfare in a triopoly (with the subsidy) to a hypothetical policy that establishes the optimal quality levels in a duopoly without triggering entry. It turns out that a well-designed policy that does not trigger entry is superior to the entry subsidy. Nevertheless, the entry subsidy may be useful in practice because it requires less information.

The fact that minimum quality standards are widely used in practice, and that their use is not restricted to duopolistic industries, appears to be at odds with the results of this chapter. Therefore, it seems plausible to assume that other sources of inefficiency, that are not related

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[111] Scarpa (1998) shows that a minimum quality standard is harmful in a vertically differentiated market with three firms. However, the author does not derive a closed-form solution and resorts to a numerical analysis. Therefore, he fails to see the generality of his result (in particular, the fact that it applies not just to minimum quality standards but to any type of regulation). He does also not identify the basic reason behind the result, namely the fact that equilibrium qualities in an unregulated market with three (or more) competitors are almost optimal.

to firms' strategic interaction in oligopoly, play an important role as well. Leland (1979) points out that "Markets which have minimum quality standards tend to be characterized by informational asymmetry, in which the seller knows the quality of his service or product, but the buyer does not.". The present chapter gives some theoretical support to this view for markets with three or more competitors.

The rest of the chapter is organized as follows. In Section 4.2, the model is introduced and solved for a varying number of firms. Section 4.3 performs the welfare analysis for a given market structure. In Section 4.4, the analysis is extended to allow for free entry. Section 4.5 concludes.

4.2. THE MODEL

Consider a market where products differ in only one dimension, called quality.[112] Suppose n firms interact in two stages.[113] In stage 1, firms simultaneously choose the quality level of their products, and in stage 2, they announce prices. On the demand side, there is a continuum of consumers with mass 1. A marginal consumer is characterized by the taste parameter t, drawn from a uniform distribution on the interval $[0,1]$. Products are sold in units, and each consumer purchases at most one unit in the market (unit demand).

Let the set of feasible quality levels be $[0,\infty)$. If the marginal consumer of type t buys a product of quality q at the price p, her surplus equals:[114]

$$u(t,q,p) = V + g(q) + tq - p \tag{1}$$

, where $g(.)$ is a non-decreasing function, and $V \geq 0$ is a constant.

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[112] In contrast to horizontal differentiation, vertical (quality-) differentiation is defined by the condition that all consumers agree upon a preference ordering of all feasible product versions. Different consumer types purchase different versions only if lower quality products are offered at a lower price.

[113] Strictly speaking, a model of this structure applies to situations in which firms choose quality only once. In a dynamic context, the two stages might be interpreted as representing one product cycle. See also Hoppe and Lehmann-Grube (2001) for an explicit analysis of the timing decision in a vertically differentiated market, and Grossman and Helpman (1991) for a dynamic game of quality growth.

[114] This is additional utility compared to the case where the consumer spends all income on other goods available in the economy. To derive (1) from a proper utility function, suppose utility is quasi-linear: $\tilde{u}(m-p,t,\hat{q}) \equiv (m-p)+\varphi(t,\hat{q})$ (m : income). Let $\varphi(t,\hat{q}) \equiv V + t \cdot h(\hat{q}) + \tilde{g}(\hat{q})$. Take $q \equiv h(\hat{q})$ as a new measure of quality, let $g(q) \equiv \tilde{g}(h^{-1}(q))$, and subtract m from the utility function (this does not affect preferences over $p-\hat{q}$ - combinations) to get (1).

According to (1), the impact of quality upon utility can be decomposed into a type-dependent part, tq, and a type-independent part, $V+g(q)$.[115] The constant V reflects utility of consuming any feasible product version, irrespective of quality. Throughout this chapter, I will assume that V is sufficiently large so that uncovered market outcomes, where some of the consumers with the lowest valuation for quality do not make a purchase, do not occur.[116] Aggregate demand, thus, always equals 1. The assumption of full market coverage simplifies the exposition considerably, but the main results would not change if the assumption is relaxed.

The marginal cost of production is assumed to be independent of quantity but increasing in quality. It is denoted by $c(q)$.[117] Profit maximizing firms serve the entire demand they face, so firm i's total production cost equals $c(q)D_i$, where D_i is demand for firm i's product. Profits are given by: $\pi_i = \left(p_i - c(q_i)\right)D_i$ ($i = 1,2,...,n$).

The above model is more general than Tirole's (1988) well-known model, and contains the latter as a special case. The relation between the two is discussed in the Appendix. The present model appears to be more realistic, and seems more suitable as a "benchmark model" for vertically differentiated markets.

Suppose, products are ordered from lowest to highest quality: $q_1 < q_2 < ... < q_n$.[118] Since consumers agree upon the ordering of products, it must hold that $p_1 < p_2 < ... < p_n$ if each firm is to sell a positive quantity. As is well-known from vertical differentiation models of this type, firm i's demand can be derived from conditions of indifference. Let t_i be the type who is indifferent between buying firm i's and firm $i+1$'s product ($i = 1,...,n-1$). The conditions of indifference read:

$$u(t_i, q_i, p_i) = u(t_i, q_{i+1}, p_{i+1}) \ , \ \ i = 1,...,n-1 \tag{2}$$

Using (1), this yields:

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[115] Type-dependent utility may also be non-linear in quality (e.g. $th(q)$). However, the measure of quality can be rescaled to yield $h(q) = q \quad \forall q$.

[116] Note, that for any set of prices and quality choices, there is a V that is sufficiently large so that even the lowest consumer type makes a purchase in the market. $V = 0$ is often sufficient for this. For a detailed discussion of covered vs. uncovered outcomes, see also Wauthy (1996), and Choi and Shin (1992).

[117] Ronnen (1991), Lehmann-Grube (1997), and Aoki (2003) assume a quality dependent fixed cost. This seems plausible when firms enter a new market.

[118] The inequalities are strict because $q_i = q_{j \neq i}$ leads to marginal cost pricing and does not occur in equilibrium.

$$t_i = \frac{p_{i+1} - p_i - g(q_{i+1}) + g(q_i)}{q_{i+1} - q_i} \tag{3}$$

Let $t_0 \equiv 0$ and $t_n \equiv 1$. Firm i's demand is, then, given by:

$$D_i = t_i - t_{i-1} \ , \ i = 1,...,n \tag{4}$$

The profit maximization problems at the pricing stage read:

$$\max_{p_i} \pi_i = \left(p_i - c(q_i)\right)\left(t_i - t_{i-1}\right) \ , \ i = 1,...,n \tag{5}$$

To simplify the exposition, I will use $c_i \equiv c(q_i)$ as a short-hand for the value of the function $c(.)$ at $q = q_i$. The short-hand is also used for other functions of q (e.g. $g_i \equiv g(q_i)$).

Furthermore, the following variable transformation turns out to be useful:

$$r_i \equiv p_i - c_i \tag{6}$$

r_i is firm i's mark-up price (price minus marginal cost). For a given value of q_i, this variable can be used instead of p_i in all computations.

It is further useful to melt the functions $g(.)$ and $c(.)$ into a single function as follows:

$$f(q) \equiv g(q) - c(q) \quad \forall q \tag{7}$$

The function $f(.)$ reflects the type-independent preference for quality net of marginal cost. To obtain an interior solution, $f(.)$ must be a concave function, and to fulfill the second-order conditions at the quality stage, $f(.)$ must be sufficiently concave. Below, a linear-quadratic form is assumed that fulfills the requirements. Note, that to assure that $f(.)$ is concave, $g(.)$ must be concave or $c(.)$ must be convex, or both. Both are plausible assumptions.

Lemma 1: The equilibrium mark-up prices r_i^* and quality choices q_i^* ($i = 1,...,n$) are independent of the exact specification of $g(.)$ and $c(.)$. They only depend on $g(.)$ and $c(.)$ via $f(.)$.

Proof:

Using (3), (6), and (7), the profit maximization problems at the pricing stage, (5), can be written as follows:[119]

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[119] Remember, that f_i stands for $f(q_i) = g(q_i) - c(q_i)$, and $r_i = p_i - c(q_i)$.

$$\max_{r_1}\left[r_1\frac{r_2-r_1-f_2+f_1}{q_2-q_1}\right] \tag{8}$$

$$\max_{r_i}\left[r_i\frac{(q_i-q_{i-1})\left(r_{i+1}-r_i-f_{i+1}+f_i\right)-(q_{i+1}-q_i)\left(r_i-r_{i-1}-f_i+f_{i-1}\right)}{(q_{i+1}-q_i)(q_i-q_{i-1})}\right],\ i=2,...,n-1$$

$$\max_{r_n}\left[r_n\frac{q_n-q_{n-1}-r_n+r_{n-1}+f_n-f_{n-1}}{q_n-q_{n-1}}\right]$$

The claim follows from the fact that equations (8) contain $g(.)$ and $c(.)$ only via $f(.)$. □

According to Lemma 1, different specifications of $g(.)$ and $c(.)$ yield the same market outcome, as long as the function $f(.)$ stays the same. This implies that market outcomes do not depend on the absolute value of marginal production costs, but only on marginal costs relative to consumers' (type-independent) willingness-to-pay for quality.

Lemma 1 has an important implication for the welfare analysis in Section 4.3. Although the equilibrium prices p_i may depend upon the exact specification of $g(.)$ and $c(.)$, welfare does not, because prices are simply transfers from consumers to firm owners and do not directly affect welfare. Similarly as the mark-up prices and quality choices, welfare will depend on $g(.)$ and $c(.)$ only via $f(.)$.

The solution to the profit maximization problems (8) is computed via the following first-order-conditions (FOC) that form a system of n linear equations in the mark-up prices:[120]

$$-2r_1+r_2=f_2-f_1 \tag{9}$$

$$(q_{i+1}-q_i)r_{i-1}-2\left(q_{i+1}-q_{i-1}\right)r_i+(q_i-q_{i-1})r_{i+1}=(q_i-q_{i-1})(f_{i+1}-f_i)-(q_{i+1}-q_i)(f_i-f_{i-1})$$

$$r_{n-1}-2r_n=-q_n+q_{n-1}-f_n+f_{n-1}$$

The FOC (9) can be substituted back into the profit maximization problems (8) to yield the following simplified expressions for profits:

$$\pi_1=\frac{r_1^2}{q_2-q_1} \tag{10}$$

$$\pi_i=\frac{r_i^2(q_{i+1}-q_{i-1})}{(q_{i+1}-q_i)(q_i-q_{i-1})},\ i=2,...,n-1$$

$$\pi_n=\frac{r_n^2}{q_n-q_{n-1}}$$

Now consider profit maximization at the quality stage. Using (10), and:

[120] It is straight-forward to check that the second order conditions are always fulfilled.

$$\frac{d\pi_i}{dq_i} = \frac{\partial \pi_i}{\partial r_i}\frac{dr_i}{dq_i} + \frac{\partial \pi_i}{\partial q_i} \ , \ i = 1,...,n \tag{11}$$

, the FOC on the quality stage can be written in the following simplified form:

$$r_1 + 2(q_2 - q_1)\frac{dr_1}{dq_1} = 0 \tag{12}$$

$$(q_{i+1} - 2q_i + q_{i-1})r_i - 2(q_{i+1} - q_i)(q_i - q_{i-1})\frac{dr_i}{dq_i} = 0 \ , \ i = 2,...,n-1$$

$$r_n - 2(q_n - q_{n-1})\frac{dr_n}{dq_n} = 0$$

Due to backwards induction, (12) can only be solved for $(q_1, q_2, ..., q_n)$ after the FOC on the pricing stage, (9), have been solved for $(r_1, r_2, ..., r_n)$. Equations (10), then, yield the equilibrium profits $\left(\pi_1^*, \pi_2^*, ..., \pi_n^*\right)$.

To obtain tractable results, from now on, a linear-quadratic form will be assumed for $f(.)$:[121]

$$f(q) = \beta q - \alpha q^2 \tag{13}$$

, where $\alpha > 0$ and $\beta > 0$ by assumption.

In the duopoly case[122], the equilibrium can easily be computed. One obtains:[123]

$$q_1^* = \frac{\beta - 1/4}{2\alpha} \ , \ q_2^* = \frac{\beta + 5/4}{2\alpha} \tag{14}$$

$$r_1^* = r_2^* = \frac{3}{8\alpha}$$

$$\pi_1^* = \pi_2^* = \frac{3}{16\alpha}$$

For $n = 3$ firms, one obtains the following solution:

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[121] Although the results in this chapter are based on this specific functional form, the main results hold more generally for a set of concave functions. Note also that, since $f(.)$ is a combination of $g(.)$ and $c(.)$, $g(.)$ and $c(.)$ are not required to be linear-quadratic, which adds some generality.

[122] For completeness, the monopoly case ($n = 1$) is briefly discussed in the Appendix.

[123] It can be shown that the necessary second order conditions are fulfilled, and that the equilibrium qualities are also global maxima. To assure market coverage, the following condition must be fulfilled (follows from (1), (6), and (7) for $t = 0$): $V \geq r_1 - \beta q_1 + \alpha q_1^2$. For $n = 2$, this yields (using (14)): $V \geq \frac{25 - 16\beta^2}{64\alpha}$. To assure market coverage, on can either assume that β is sufficiently large, or that $V > 0$.

$$q_1^* = \frac{\beta + 1/8}{2\alpha} \ , \ q_2^* = \frac{\beta + 1/2}{2\alpha} \ , \ q_3^* = \frac{\beta + 7/8}{2\alpha} \tag{15}$$

$$r_1^* = r_3^* = \frac{13}{256\alpha} \ , \ r_2^* = \frac{11}{256\alpha}$$

$$\pi_1^* = \pi_3^* = \frac{169}{12288\alpha} \ , \ \pi_2^* = \frac{121}{12288\alpha}$$

For $n \geq 4$, computations become so complicated even with the help of a computer that the problem must be simplified. This can be achieved by assuming that the solution has certain properties, which allows to reduce the number of variables in the computation. The inspection of the above results for $n = 2$ and $n = 3$ reveals that equilibrium qualities are located symmetrically around $\frac{\beta + 1/2}{2\alpha}$, and are of the form $q_i = \frac{\beta + x_i}{2\alpha}$, where the $x_i \in \mathbb{R}$ depend only on the number of firms, n, and are independent of all other parameters of the model. The values x_i may be referred to as *reduced quality levels*:

$$x_i \equiv 2\alpha q_i - \beta \tag{16}$$

Assuming that the above assumptions hold for $n > 3$, candidate equilibria can be computed for any given number of firms. It must, then, be verified that each of them fulfills the necessary and sufficient equilibrium conditions. Using this approach, I have solved the problem for up to $n = 7$ firms. The equilibrium reduced quality levels are shown in Figure 1.

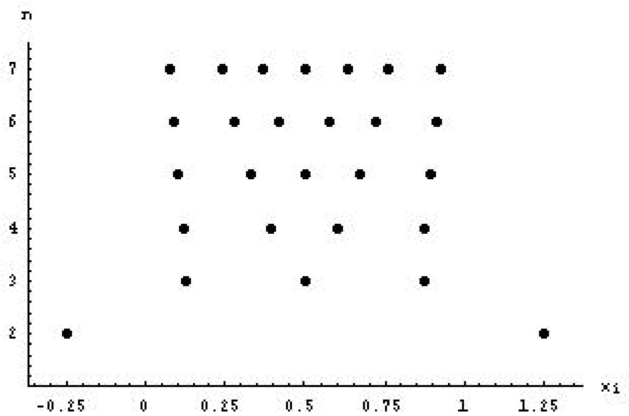

Figure 1: Equilibrium reduced quality levels x_i for a varying number of firms n

Figure 1 illustrates that a "regime change" occurs at the transition from $n = 2$ to $n \geq 3$, as the range of quality levels is narrowed remarkably at this transition (and gradually increases again for larger values of n). This is accompanied by a dramatic decline in profits (industry profit decreases by roughly factor 10 at the transition from $n = 2$ to $n = 3$). The presence of a regime change suggests that results or policy prescriptions that are derived for duopolistic markets may not hold for markets with more than two competitors.

Range of preferred quality levels: It is instructive to compare the range of equilibrium quality levels ($q_n^* - q_1^*$) with the range of quality levels preferred by consumers. Let $\tilde{q}(t)$ be the preferred quality level of the marginal consumer of type t when all feasible quality levels are available to the consumer at marginal cost. Use $p = c(q)$ in (1), and maximize over q to obtain the following optimality condition:

$$t + \frac{df(\tilde{q})}{dq} = 0 \tag{17}$$

(17) is easy to interpret. t is the type-specific marginal utility with respect to an increase in quality, and $\frac{df(\tilde{q})}{dq}$ is type-independent marginal utility net of marginal cost. Optimality requires the overall benefit of a marginal increase in quality to be zero.
Using (13), (17) yields:

$$\tilde{q}(t) = \frac{\beta + t}{2\alpha} \tag{18}$$

Plugging in $t = 0$ for the lowest, and $t = 1$ for the highest consumer type, we, thus, obtain for the range of preferred quality levels: $\left[\frac{\beta}{2\alpha}, \frac{\beta+1}{2\alpha}\right]$. Note, that $\frac{\beta + 1/2}{2\alpha}$, the mid-point of equilibrium qualities for $n \geq 2$, is also the center of the interval of preferred qualities.
Comparing the range of equilibrium quality choices in a market equilibrium (see Figure 1) with the range of preferred quality levels, we find that for $n \geq 3$, equilibrium qualities are located at the interior of the range of preferred quality levels, whereas in the duopoly case, they are located outside. This illustrates that, in an attempt to soften price competition, duopolistic firms differentiate so much in the quality dimension that equilibrium qualities are not optimal qualities for any consumer type. There is, thus, a high degree of over-differentiation in the duopoly case that can cause a substantial welfare loss.
The reason why equilibrium qualities are less dispersed as soon as more than two competitors are in the market is the following. If, say, firm 1 in the duopoly case lowers its quality choice in stage 1, this will soften firm 2's pricing behavior in the following stage.[124] This gives firm 1 an incentive to choose an artificially low quality level. When there are three firms in the

[124] Reaction functions shift towards higher prices.

market, firm 1 has two competitors who are also engaged in a price competition among each other. This implies that firm 2 is now less responsive to a quality reduction by firm 1. Therefore, firm 1 has a weaker incentive to differentiate away from its neighboring competitor. The same reasoning applies to firm 3, so overall, equilibrium qualities are less dispersed than in a duopoly. Although the quantitative results (in particular the welfare analysis conducted in Sections 4.3 and 4.4) rely on the specific modeling framework chosen in this chapter, the basic finding of a regime change in the nature of competition at the transition from $n = 2$ to $n = 3$ or more firms is of a much more general nature, so the prediction that qualities are less dispersed than in a duopoly, and the consequences that are derived from this, are robust.

The over-differentiation result obtained in the duopoly case, and the welfare loss it causes has inspired authors to analyze the effects of different policy measures on welfare in vertically differentiated markets. As pointed out above, the presence of a regime change at the transition from duopoly to triopoly raises concerns about the generality of the results. Since the nature of competition in vertically differentiated markets with more than two firms differs from the one in the duopoly case, we can not expect conclusions that were obtained for the duopoly case to be applicable to markets with than two competitors. The following two Sections analyze the scope for welfare improvements by regulation for a varying number of firms. It is shown that, as soon a more than two competitors are in the market, outcomes are almost optimal, which leaves little room for welfare improving policies.

4.3. WELFARE ANALYSIS

To compare welfare in a market equilibrium with the optimum (the social planner's case) for a varying number of firms, I will first derive an expression that allows to compute welfare for a given set of quality levels and mark-up prices.

I assume consumer surplus (S) and industry profit (Π) enter welfare in an identical way:

$$W = S + \Pi \tag{19}$$

Consumer surplus is computed by summing over the different products and integrating over consumer types, taking into account consumers' product choices:

$$S_n = \sum_{i=1}^{n} \left[\int_{t_{i-1}}^{t_i} u(t, q_i, p_i) dt \right] \tag{20}$$

Using (1), (6), and (7), and evaluating the integral, this can be written as:

$$S_n = V + \frac{1}{2} \sum_{i=1}^{n} \left[(t_i - t_{i-1}) \left(2f(q_i) - 2r_i + q_i (t_i + t_{i-1}) \right) \right] \tag{21}$$

Industry profit is simply the sum over the n firms' profits. Using (4) and (6), we obtain:

$$\Pi_n = \sum_{i=1}^{n} \pi_i = \sum_{i=1}^{n} r_i (t_i - t_{i-1}) \tag{22}$$

Using (21) and (22) in (19), we obtain the following expression for welfare:

$$W_n = V + \frac{1}{2}\sum_{i=1}^{n}\left[(t_i - t_{i-1})\left(2f(q_i) + q_i(t_i + t_{i-1})\right)\right] \tag{23}$$

The values for t_i are computed as (follows from (3) and (6)):

$$t_i = \frac{r_{i+1} - r_i - f(q_{i+1}) + f(q_i)}{q_{i+1} - q_i} \tag{24}$$

Note, that prices affect welfare only via market shares (that are determined by the t_i's). This is because prices are simply transfers from consumers to firm owners and, thus, have no direct effect upon welfare. Market shares and quality levels, however, affect welfare directly. Inefficiencies in vertically differentiated markets can, thus, arise due to distorted quality levels, or due to distortions in market shares, which implies that some consumers do not purchase from the same firm where they would buy in the planner's case (or both).[125]

The Social Planner's case:

The planner's problem is to maximize welfare by choosing prices and qualities optimally. Since welfare depends upon prices only via the t_i's, the problem can be decomposed into two steps: 1. determine the vector $(q_1,...,q_n,t_1,...,t_{n-1})$ that maximizes (23) for a given n, and 2. choose mark-up prices r_i such that (24) is fulfilled $\forall i \in \{1,...,n-1\}$. Note, that in step 2, the regulator has one variable more than necessary to fulfill the $n-1$ conditions, because there are n mark-up prices to choose. Intuitively, suppose the planner chooses for product 1 some price greater than marginal cost ($r_1 > 0$). This does not cause an inefficiency if the planner chooses the remaining prices and all quality levels such that the optimal allocation is reached. Now all mark-up prices are greater than zero. Therefore, if the planner values consumer surplus only marginally more than profits, the optimal choice will always be $r_1 = 0$, and, not surprisingly, all other mark-up prices will optimally be zero as well. However, rather than imposing this, this should be the result of the planner's optimization problem, because the

[125] Another potential source of inefficiency arises in uncovered market situations when too many consumers (as compared to the planner's case) do not make a purchase. As pointed out in Section 4.2, this chapter focuses on covered market outcomes, so this possibility does not arise.

planner may set prices different from marginal costs to correct for other inefficiencies. Therefore, we are only free to choose one mark-up price.[126] I, thus, assume $r_1 = 0$.

Using (13) in (23), the first step of the planner's problem reads:

$$\max_{q_1,\dots,q_n,t_1,\dots,t_{n-1}} \sum_{i=1}^{n} \left[(t_i - t_{i-1}) \left(2\beta q_i - 2\alpha q_i^2 + q_i (t_i + t_{i-1}) \right) \right] \tag{25}$$

The FOC can be written as follows:

$$t_i = \alpha(q_{i+1} + q_i) - \beta \quad , \quad i = 1,\dots,n-1 \tag{26}$$

$$0 = 2\beta - 4\alpha q_i + t_i + t_{i-1} \quad , \quad i = 1,\dots,n \tag{27}$$

Plugging the t_i's from (26) into (27), we obtain the following relation:

$$q_i = \frac{q_{i+1} + q_{i-1}}{2} \quad , \quad i = 2,\dots,n-1 \tag{28}$$

(28) illustrates that, in the optimum, there are equal distances between all quality levels.

To solve the system of linear equations (26) and (27) for an arbitrary number of firms n, the easiest approach is to guess (part of) the solution, and then to verify that the candidate solution fulfills the equations. Since quality levels are equally spaced, an obvious guess is that all market shares are identical in the planner's solution. I, thus, assume:

$$t_i = \frac{i}{n} \quad , \quad i = 1,\dots,n-1 \tag{29}$$

Plugging this into (27), we obtain for the planner's quality choices:

$$q_i = \frac{\beta + \frac{i-1/2}{n}}{2\alpha} \quad , \quad i = 1,\dots,n \tag{30}$$

All that remains to be done is to use (29) and (30) in (26) to show that the candidate solution is the correct solution to step 1 of the planner's problem (this is straight-forward).

Step 2 of the planner's problem is to choose mark-up prices that fulfill (24). Solving (24) for r_{i+1}, we obtain (using (29) and (30)):

$$r_{i+1} = r_i \quad , \quad 1,\dots,n-1 \tag{31}$$

All mark-up prices are, thus, identical, which, combined with the assumption $r_1 = 0$, yields $r_i = 0$, $i = 1,\dots,n$. The planner, thus, sells all product versions at marginal cost.

Using the above results, welfare in a market equilibrium can be compared with the respective

[126] Furthermore, this mark-up price must be sufficiently low to yield market coverage.

planner's outcome for an arbitrary number of firms. For the purpose of this chapter, it suffices to show the results for $n=2$ and $n=3$ firms:

$$W_{n=2}^{sp} = V + \frac{\beta^2 + \beta + 5/16}{4\alpha} \quad , \quad W_{n=2}^{*} = V + \frac{\beta^2 + \beta + 1/16}{4\alpha} \tag{32}$$

$$W_{n=3}^{sp} = V + \frac{\beta^2 + \beta + 35/108}{4\alpha} \quad , \quad W_{n=3}^{*} = V + \frac{\beta^2 + \beta + 989/3072}{4\alpha} \tag{33}$$

Note, that the welfare reductions in the market equilibrium relative to the planner's case result for different reasons. For $n=2$, both firms' market shares are equal to 1/2 in the planner's case and in the market equilibrium. The lower welfare in the market equilibrium is, thus, due to the fact that qualities are too dispersed (over-differentiation). For $n=3$, qualities in the market equilibrium are (slightly) too dispersed, and in addition, market shares are skewed[127], which implies that there is a misallocation of consumers over the firms (some consumers purchase from a different supplier than in the planner's case).

Note, that all expressions for welfare in (32) and (33) are of the same form: $W = V + \frac{\beta^2 + \beta + const}{4\alpha}$. This permits quantitative predictions about potential welfare improvements by regulation. Consider the following measure for the maximum percentage welfare increase that a regulator can achieve by setting prices and qualities optimally (for a given number of firms n):

$$R_n \equiv \max_{V,\alpha,\beta} \left[\frac{W_n^{sp} - W_n^{*}}{W_n^{*}} \right] \tag{34}$$

R_n is, thus, an upper bound to the percentage welfare increase a regulator can achieve, which means that we can not find parameter values that yield a higher relative welfare increase. It is a measure of the *scope for welfare improvements* that can be achieved by regulation.

Computing the ratio $\frac{W_n^{sp} - W_n^{*}}{W_n^{*}}$ for a given value of n, we find that the parameters V and β drop out in the numerator, and in the denominator, they appear with positive signs. This implies that the ratio strictly decreases in V and β. Therefore, to compute an upper bound to the percentage welfare increase, simply set $V = \beta = 0$.[128] The parameter α, then, drops out.

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[127] The high- and the low-quality provider have a market share of about 27%, and the medium firm of 46%. Mark-up prices are also not identical (see (15)).

[128] A parameter restriction of the form $\beta \geq \underline{\beta} > 0$ or $V \geq \underline{V} > 0$ is sometimes necessary to assure non-negativity of the lowest equilibrium quality and/or market coverage. These restrictions are for simplicity neglected in the

Evaluating (34) for the duopoly case, we obtain an upper bound to the percentage welfare increase of 200 percent. This implies that, depending on the actual parameter values, there can be a substantial scope for welfare improving policy interventions in the duopoly case. Results are, however, markedly different when the number of competitors is larger. The main result of this Section is summarized by the following Proposition:

Proposition 1: When three firms are in the market, there is an upper bound to the percentage welfare increase by regulation of only 0.66 percent.

Proof:

Using (33) and (34), we obtain for $V = \beta = 0$: $R_3 = \dfrac{35/108 - 989/3072}{989/3072} \cong 0.0066$.

According to Proposition 1, when there are three firms in the market, the scope for welfare improving policies is small.[129] It should be pointed out that for any plausible parameterization of the model ($\beta > 0$), the actual scope for welfare improvements is even smaller, as the upper bound has been computed for extreme parameter values.

The reason for behind the result is that the equilibrium qualities when three (or more) firms are in the market are close to the optimum, as can easily be confirmed by comparing (15) and (30) (for $n = 3$). The over-differentiation result that drives the inefficiencies in the duopoly case, thus, almost disappears at the transition from $n = 2$ to $n = 3$ or more firms. Furthermore, even if the specific value for the upper bound to potential welfare improvements that was computed above relies on the specific modeling choices made, the main result, namely that the scope for welfare improvements decreases dramatically at the transition from $n = 2$ to $n = 3$ firms, holds more generally and does not depend on the specific modeling choices made (such as the linear-quadratic specification of the function $f(.)$).

As a consequence, we can conclude that a *laissez-faire* policy seems appropriate whenever 3 or more firms are in the market. This is especially true if the regulator faces some uncertainty

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computation of the upper bound to potential welfare increases. However, in both cases, ($\beta > 0$ or $V > 0$), incorporating the restriction into the computation would only lead to a *reduction* in the upper bound, thus strengthening the main results of the analysis further.

[129] This holds *a fortifiori* for an even larger number of firms since prices and quality choices are even closer to the planner's solution.

about the true parameter values of the model (as is always the case in reality). A policy that is intended to increase welfare may, then, even turn out to be harmful.

4.4. WELFARE UNDER FREE ENTRY CONDITIONS

In this section, the analysis is extended to allow for free entry. Let $F \geq 0$ be the fixed entry cost per firm. The main questions that shall be answered are whether the result of Proposition 1 continues to hold under free entry conditions, and whether a regulator can use a subsidy to entry to improve welfare.

As a benchmark, consider first a situation with zero entry costs. To maximize welfare, the social planner offers a continuum of product qualities (the interval between the lowest and the highest preferred quality) at marginal cost, and each consumer type chooses her preferred quality level $\tilde{q}(t)$. Using (1), (7), (13), and (18), we obtain for the utility of type t:

$$u^{\infty}(t) = V + \frac{(\beta + t)^2}{4\alpha} \tag{35}$$

Welfare, thus, equals:

$$W^{\infty} = \int_0^1 u^{\infty}(t)dt = V + \frac{\beta^2 + \beta + 1/3}{4\alpha} \tag{36}$$

This is the maximum welfare that can be achieved in this market in the absence of entry costs. This formula can be used to compute a simple estimate for an upper bound to welfare improvements by any type of regulation. Note, that, under free entry conditions, the market equilibrium with zero entry costs coincides with the planner's solution: an infinite number of firms enter, and welfare is as in (36).

In the presence of entry costs, the optimal number of firms is finite. (36), then, always overestimates the scope for welfare improvements by regulation. When 3 firms are in the market, using (33) and (36), we obtain as an estimate of the scope for welfare improvements: $\frac{W^{\infty} - W_3^*}{W_3^*} = 3.5$ percent. This illustrates that the market solution with 3 firms leaves rather little scope for welfare improvements even when compared to the first best solution with an infinite number of product versions where entry costs are neglected.

When entry costs are taken into account, the scope for welfare improvements is smaller. The following Proposition extends the result of Proposition 1, taking into account entry costs:

Proposition 2: When three firms are in the market, the upper bound to the percentage welfare increase by any regulation that leaves the number of competitors unchanged equals 1.6 percent.

Proof:

When entry costs are taken into account, the formula for welfare (19) becomes:

$$W = S + \Pi - nF \tag{37}$$

, where Π is industry profit before entry costs.

Suppose, entry occurs in stage 0, before firms coordinate on their positions in the quality dimension. Therefore, entry occurs as long as the following condition holds:

$$\Pi(n) - nF \geq 0 \tag{38}$$

To obtain an upper limit to potential welfare improvements, we want to compute R_n as in (34), but now the maximization is also over F, and we have to subtract the total entry cost nF from all expressions for W. In the numerator, entry costs drop out, while the denominator decreases in F. R_n is, thus, maximized when F is maximized. Since 3 firms are in the market, we know that $F \leq \frac{\Pi(3)}{n}$ holds. Therefore, to maximize R_n, let $F \equiv \frac{\Pi(3)}{n}$. Industry profit, then, equals zero, so welfare equals consumer surplus. For $n = 3$, this is given by:

$$S^*_{n=3} = V + \frac{\beta^2 + \beta + 409/3072}{4\alpha} \tag{39}$$

Using (33) and (39), we obtain for $V = \beta = 0$: $R_3 = \frac{35/108 - 989/3072}{409/3072} \cong 0.016$.

The difference between the results of Propositions 1 and 2 can be interpreted as follows. When fixed entry costs are taken into account, profits are partially or fully dissipated, so total welfare is reduced, while the difference in welfare with / without regulation stays the same. This yields a slightly higher estimate of the scope for welfare improvements measured in relative terms. However, the basic result, that the scope for welfare improvements is small when 3 firms are in the market, remains valid.

It has been shown that, when 3 or more firms are in the market, there is little scope for welfare improvements, so a laissez-faire policy seems generally appropriate. The following results, thus, focus on the duopoly case where (depending on the parameter values) substantial welfare improvements may be possible. In particular, I will analyze whether a subsidy to entry may be welfare improving, which seems plausible as welfare is almost maximized for

$n = 3$. Note, that a subsidy to entry has (compared to other policies) the advantage that the regulator does not need to influence the firms' choices of qualities and prices directly.

Proposition 3: When two firms are in the market, there is an interval of fixed entry costs F where entry is welfare improving but does not occur under market conditions. A subsidy to entry is, thus, welfare improving.

Proof:

Under free entry conditions, at most two firms enter the market if $\Pi(3) - 3F < 0$. Using (15), this yields the condition:

$$F > \frac{51}{4096\alpha} \tag{40}$$

Starting from a situation with two firms, entry by a third firm leads to the following change in welfare (using (32) and (33)):

$$W_3^* - W_2^* - F = \frac{797}{12288\alpha} - F \tag{41}$$

Therefore, for $\frac{51}{4096\alpha} < F < \frac{797}{12288\alpha}$, the entrant stays out, although entry would be desirable from a welfare perspective.

The result of Proposition 3 stems from the fact that there is insufficient entry under market conditions. This result is in contrast to standard oligopoly theory (e.g. Cournot competition), where prices in excess of competitive prices usually induce more entry than optimal, due to an inefficient replication of entry costs. In the present model, there is a substantial increase in consumer surplus at the transition from $n = 2$ to $n = 3$ because over-differentiation almost vanishes, while profits decrease sharply.[130] This leads to insufficient entry.

Note, that, if a subsidy is used to trigger entry, it must be at least as high as $F - \frac{51}{4096\alpha}$ to make the entrant's profit non-negative. A subsidy should not be granted if $F > \frac{797}{12288\alpha}$. If the actual level of the fixed entry cost F is unknown to the regulator, the optimal policy is to

[130] Note, that, unlike in the present model where a regime change occurs at the transition from $n = 2$ to $n = 3$, in Cournot markets, the transition to competitive pricing as the number of firms increases is a gradual one.

offer a subsidy of $\frac{797}{12288\alpha} - \frac{51}{4096\alpha} = \frac{161}{3072\alpha}$ to a potential entrant, since entry will, then, occur if and only if it is socially desirable.

The result of Proposition 3 raises the question where to look for an optimal policy. In the following, I want to answer the question whether a subsidy to entry is superior to a hypothetical optimal policy that leaves the number of competitors unchanged (starting from a situation with two firms in the market). The result is summarized in Proposition 4.

Proposition 4: If two firms are in the market, a subsidy to entry is inferior to an optimal policy that leaves the number of competitors unchanged.

Proof:

A subsidy to entry is superior to an optimal policy that leaves the number of firms unchanged if $W_3^* - F > W_2^{sp}$, where W_2^{sp} is welfare in the planner's case for $n = 2$. Using (32) and (33), this yields the condition:

$$F < \frac{29}{12288\alpha} \tag{42}$$

However, by (40), when there are two competitors in a market, we must infer that entry costs are at least as high as $\frac{51}{4096\alpha}$. Therefore, condition (42) is violated.

Proposition 4 implies that a policy that installs optimal prices and qualities in the duopoly case, without triggering entry, is always superior to a subsidy to entry. Nevertheless, there are arguments in favor of an entry subsidy. If welfare is to be increased by means of taxation, minimum quality standards, or other policies that aim at establishing quality choices that are close to the optimum, then the regulator must have sufficient knowledge about the demand and cost structure in the market, which enables him to compute the optimum. A subsidy to entry, on the other hand, does not require as much information. The welfare enhancing effect of this policy relies strictly on *competitive forces*, and as long as the subsidy is not too high, it will always be welfare improving.

4.5. Concluding Remarks

A well-established result for duopoly models with vertically differentiated products and fully informed consumers is that firms differentiate too much in the quality dimension in order to soften price competition (over-differentiation). This should not be confused with the "principle of differentiation" (Tirole, 1988), according to which firms do not choose identical positions in the quality space in order to avoid marginal cost pricing. While the principle of differentiation seems fairly robust, the over-differentiation result turns out to be very sensitive to the specific modeling choices. In particular, the present chapter showed that over-differentiation almost disappears when three or more competitors (instead of two, as assumed in previous works) are in the market. Since qualities are close to the optimum, there is little scope for welfare improving policies. Policies that can improve welfare in a duopoly (such as minimum quality standards) may, thus, not be suitable for markets with three or more competitors.

Since in practice, minimum quality standards are widely used, and their use is *not* restricted to duopolies, this indicates that there may be other sources of inefficiencies that are not related to strategic over-differentiation in vertically differentiated duopolies. An obvious candidate is incomplete consumer information, e.g. related to unobservable quality features of products before or even after purchase. To name only two examples, consider the safety features of a car that are unobservable at the time of purchase, or potentially harmful ingredients in food.

For vertically differentiated markets with *well-informed consumers*, the welfare analysis conducted in the present chapter allows the following policy prescriptions:

1. When two competitors are in the market, a subsidy to entry is welfare enhancing if entry costs are not too high.
2. When three or more competitors are in the market, a laissez-faire policy should be adopted.

4.6. APPENDIX

The relation to Tirole's (1988) model of vertical differentiation:

Tirole's (1988) well-known vertical differentiation model is a special case of the model introduced in this chapter. Tirole assumes: $u = \vartheta q - p$, where the preference parameter ϑ is distributed uniformly on the interval $[\underline{\vartheta}, \overline{\vartheta}]$. This expression for u is obtained when $g(.)$ is linear: $g(q) \equiv \beta q$, and $V = 0$. (1), then, reads: $u = \left(t + \beta\right) q - p$. Define $\vartheta \equiv t + \beta$. Since t is uniform on $[0,1]$, ϑ is uniform on $[\underline{\vartheta} \equiv \beta, \overline{\vartheta} \equiv \beta + 1]$. Arbitrary values for $\underline{\vartheta}$ and $\overline{\vartheta}$ are reached by adjusting β and by appropriately rescaling the measure for quality by a constant factor. Note, that a larger value of β corresponds to less dispersed preferences in Tirole's formalization. Tirole further assumes that costs are independent of quality, which seems restrictive.

Tirole points out that the "types" can be re-interpreted as consumers with different levels of income. If $p << m$, a first-order Taylor expansion of utility yields: $\tilde{U}(m-p,\hat{q}) \cong \tilde{U}(m,\hat{q}) - (\partial\tilde{U}/\partial m)p$. Let $\tilde{U}(m,\hat{q})$ be given by $m^{\alpha} + h(\hat{q})$ (the surplus derived from consuming one unit of a product of quality $\hat{q}$ is the same for all consumers). Subtract m^{α} from the utility function and divide by $\partial\tilde{U}/\partial m = \alpha m^{\alpha-1}$ to get $\hat{U}(m,p,\hat{q}) = m^{1-\alpha} \cdot h(\hat{q})/\alpha - p$. If m has the distribution function $F(m) = m^{1-\alpha} - \beta$ over the interval $[\beta^{1/(1-\alpha)}, (1+\beta)^{1/(1-\alpha)}]$, we get $U(t,p,\hat{q}) = (t+\beta)h(\hat{q})/\alpha - p$, where $t \equiv m^{1-\alpha} - \beta$ is distributed uniformly over $[0,1]$. Now take $q \equiv h(\hat{q})/\alpha$ as a new measure of quality. Since $h(.)$ should normally be a concave function, $c(.)$ automatically becomes (more) convex (if $c(.)$ is linear: $c(\hat{q}) = \chi\hat{q}$, then $c(q) = \chi h^{-1}(\alpha q)$). This important fact has been ignored by Tirole, who suggests to linearise the quality-dependent part of utility, but assumes that c is independent of q, which seems restrictive. The non-linear model presented in this chapter seems more realistic, and it produces interior covered market solutions.

A brief discussion of the monopoly case:

Under market coverage, demand equals 1, so the monopoly profit is simply equal to the mark-up price of the monopolist: $\pi^m = r$. Profit is, thus, increasing in r, so the monopolist either chooses r such that the lowest consumer type is indifferent between buying and not buying (corner solution), or an uncovered market outcome is obtained. The corner solution is characterized by:

$$u(t=0,q,p) = V + g(q) - p = 0 \tag{43}$$

Using (6), this yields:

$$r = V + f(q) \tag{44}$$

Maximizing this over q, we obtain the following monopoly outcome:

$$\begin{aligned} q^m &= \frac{\beta}{2\alpha} \\ \pi^m &= r^m = V + \frac{\beta^2}{4\alpha} \end{aligned} \tag{45}$$

To assure that a covered market equilibrium is obtained, it suffices to assume that the value V is sufficiently large.[131]

[131] To see this, write down the profit for an uncovered market for a given q, and take the derivative w.r.t. r. Evaluate this at $r = V + f(q)$, the price where the lowest type is indifferent between buying and not buying. If

Welfare in the monopoly case and in the respective planner's case is given by:

$$W_{n=1}^{sp} = V + \frac{\beta^2 + \beta + 1/4}{4\alpha} \quad , \quad W_{n=1}^{*} = V + \frac{\beta^2 + \beta}{4\alpha} \tag{46}$$

The welfare loss in a monopoly, as compared to the planner's case, results from the under-provision of quality (the monopolist chooses the quality level that is preferred by the consumer with the lowest preference for quality, whereas the planner chooses the average consumer's preferred quality.) A minimum quality standard seems an appropriate measure to improve the market outcome.

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the derivative is non-positive (the condition for this is $V \geq q - f(q)$), the monopolist does not benefit from increasing his price, so the market remains covered. This holds for any given q if V is sufficiently large.

Chapter 5. ON THE ROBUSTNESS OF THE HIGH-QUALITY ADVANTAGE UNDER VERTICAL DIFFERENTIATION

5.1. INTRODUCTION

A vertically differentiated market is a market where consumers agree on the ranking of all feasible product types when they are sold at the same price.[132] In a duopolistic market with fully informed consumers, where each firm sells one product of a certain quality[133], firms never choose the same quality level because price competition then drives profits to zero. In order to characterize the nature of competition in vertically differentiated markets, one must, thus, understand how firms choose the quality of their products in equilibrium, given that they compete also in prices, and what this implies for the resulting profits. It appears to be an established fact that the high-quality provider earns the higher profit. This chapter questions this result and shows that it relies on the specific modeling choices made by various authors.

Tirole (1988), e.g., introduces a model where utility is linear in quality, and unit cost is independent of quality.[134] He shows that the high-quality provider always earns the higher profit. Lehmann-Grube (1997) shows that the high-quality-advantage is still obtained when there are (convex) fixed costs of implementing a desired quality level. The high-quality advantage, thus, seems to be a robust prediction. Unfortunately, the author does not offer an explanation of what actually drives the high-quality advantage, which makes it difficult to question the robustness of his prediction even on a qualitative level. The mere fact that the high-quality provider sells to the higher-willingness-to-pay consumers is not a convincing argument for the generality of the prediction when costs are increasing in quality.

Crampes and Hollander (1995) introduce a model where firms earn equal profits in equilibrium when the unit cost of production is quadratic in quality. This already implies that the strict high-quality advantage can not be a general prediction, but it leaves open the question under what conditions and why a high- or a low-quality advantage is obtained.

This chapter fills the gap between Tirole's (1988) linear and the quadratic version of Crampes and Hollander's (1995) model. Based on the assumption that either utility is concave or unit

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[132] When products are sold at unit cost, different consumer types may prefer different quality levels. Otherwise, there is an upper bound to the number of active firms in the market (see Shaked and Sutton, 1982 and 1983).

[133] Champsaur and Rochet (1989) analyze a duopolistic market where firms offer an interval of qualities.

[134] Tirole's model has become popular in the literature and has been used or modified by several authors, e.g. Wauthy (1996), Hoppe and Lehmann-Grube (2001), Choi and Chin (1992).

cost is convex in quality, or both, interior covered market equilibria are obtained. It is shown that, for a low degree of non-linearity, the low-quality provider earns the larger profit, while for higher degrees of non-linearity, the high-quality advantage is restored. The intuition behind this result is as follows. For a high degree of non-linearity, the high-quality provider is effectively committed not to choose very high quality levels as the difference between consumers' willingness-to-pay and unit cost decreases rapidly in quality at high quality levels. Anticipating the competitor's reluctance to choose high quality levels, the low firm has a larger incentive to choose a lower quality level, which benefits the high firm. The resulting high-quality advantage is, thus, a consequence of the firms' strategic interaction. For a low degree of non-linearity, the situation is reversed, and a low-quality advantage is obtained.

Tirole (1988) imposes exogenous bounds to the set of feasible quality choices. The upper bound is necessary because the high firm always has an incentive to increase quality, while the lower bound assures the existence of a covered market equilibrium[135]. However, exogenous bounds to quality affect the firms' strategic interaction. An upper bound has a similar effect as a (sufficiently) concave utility – quality, or a convex unit cost - quality relation. The high-quality provider is, then, committed not to choose very high quality levels.

In the model presented in this chapter, the assumption of an upper or a lower bound to quality is not necessary. If an exogenous upper bound is nevertheless introduced, it can induce an artificial high-quality advantage that might otherwise not be observed, while a lower bound to quality can induce a low-quality advantage. The chapter is meant to contribute to our understanding of where a high-quality advantage comes from, in case it is observed, and to clarify that its existence is not a general prediction for vertically differentiated markets.

5.2. THE MODEL

The model analyzed in this chapter differs from the one introduced in chapter 4 in two respects. Firstly, the analysis in this chapter is restricted to the duopoly case ($n = 2$). And secondly, a more general non-linear dependency between willingness-to-pay minus cost and quality is introduced (see below). In chapter 4, the focus was on the special case of a quadratic dependency. Since the basic model is the same, the main assumptions and results that are relevant for this chapter are, in the following, only briefly summarized.

Two firms (a high- and a low-quality provider) interact in two stages. In stage 1, they choose the quality level of their products (q_1 and q_2), and in stage 2, they announce prices (p_1 and

[135] "covered" means that all consumer types buy a product in this market

p_2). There is a continuum of consumers with mass one. A marginal consumer is characterized by the taste parameter t, that is uniformly distributed on $[0,1]$. A consumer buys either zero or one unit in the market. If type t buys a product of quality q at price p, her utility equals:

$$u(t,q,p) = V + g(q) + tq - p \tag{1}$$

, where $g(.)$ is a non-decreasing concave function, and $V \geq 0$ is a constant.

The unit cost of production is independent of the produced quantity, but increasing and convex in quality. It is denoted by $c(q)$.

To simplify notation, c_i and g_i are used as a short-hand for $c(q_i)$ and $g(q_i)$. It is further useful to define:

$$f(q) \equiv g(q) - c(q) \tag{2}$$

$f(q)$ is type-independent surplus net of cost from consuming a good of quality q.

The derivation of the equilibrium profits on the pricing stage (for fixed quality levels) is as in chapter 4. The equilibrium profits in the duopoly case are:

$$\pi_1 = \frac{(q_2 - q_1 - f_2 + f_1)^2}{9(q_2 - q_1)} \quad \text{and} \quad \pi_2 = \frac{(2(q_2 - q_1) + f_2 - f_1)^2}{9(q_2 - q_1)} \tag{3}$$

There are two conditions that assure non-negative profits that follow from (3):

$$q_2 - q_1 - f_2 + f_1 \geq 0 \quad \text{and} \quad 2(q_2 - q_1) + f_2 - f_1 \geq 0 \tag{4}$$

While in Tirole's linear model, π_2 is unambiguously larger than π_1 when both firms are active – this is the so-called high-quality advantage (see Wauthy, 1996) – this must not be the case here. It holds that $\pi_1 > \pi_2$ if:

$$2(f_2 - f_1) + q_2 - q_1 < 0 \tag{5}$$

Note, that (5) can not be fulfilled if $c_1 = c_2$. Therefore, a low-quality advantage can only be generated in a covered market situation if the unit cost is increasing in quality.[136]

The first-order-conditions at the *quality stage* read:

$$2f'(q_1) = \frac{q_2 - q_1 + f_2 - f_1}{q_2 - q_1} \quad \text{and} \quad 2f'(q_2) = \frac{-2(q_2 - q_1) + f_2 - f_1}{q_2 - q_1} \tag{6}$$

According to (6), $f(.)$ must be a concave function if it is to produce an interior covered market solution. This is fulfilled since, by assumption, $c(.)$ is convex and $g(.)$ is concave.

[136] It can be shown that the firm that earns the higher profit in equilibrium (firm $i \in \{1,2\}$) always serves more than half of the market ($D_i > 1/2$) and has a higher mark-up price ($p_i - c_i > p_{-i} - c_{-i}$).

Computing the second derivatives of the profits from (3), and using (6), one obtains the following (necessary) second-order-conditions:

$$-f''(q_1) \geq \frac{q_2 - q_1 - f_2 + f_1}{4(q_2 - q_1)^2} \quad \text{and} \quad -f''(q_2) \geq \frac{2(q_2 - q_1) + f_2 - f_1}{4(q_2 - q_1)^2} \tag{7}$$

By (4), the right-hand sides are non-negative. $f(.)$ must, thus, be sufficiently concave in the relevant range to produce an interior covered market solution.

5.3. HIGH-QUALITY ADVANTAGE VS. LOW-QUALITY ADVANTAGE

To analyze the impact of the "degree of non-linearity" of $f(.)$ upon the relation between the two firms' profits, the following specification of $f(.)$ is used:

$$f(q) = \beta q - \alpha q^{\delta} \tag{8}$$

, where α and β are non-negative constants, and $\delta > 1$.[137]

Before analyzing the equilibrium under market conditions, it is instructive to characterize the preferred quality level $\tilde{q}(t)$ of consumer type t, given that price equals unit cost. As shown in chapter 4, $\tilde{q}(t)$ is defined by the condition:

$$t + \frac{df(\tilde{q})}{dq} = 0 \tag{9}$$

For $\delta = 2$, (9) yields: $\tilde{q}(t) = \dfrac{\beta + t}{2\alpha}$. The range of preferred qualities is, thus $\left[\dfrac{\beta}{2\alpha}, \dfrac{\beta + 1}{2\alpha}\right]$.[138]

Under market conditions, the equilibrium quality levels in the quadratic case are:

$$q_1^* = \frac{\beta - 1/4}{2\alpha} \;,\; q_2^* = \frac{\beta + 5/4}{2\alpha} \tag{10}$$

According to the "principle of differentiation" (Tirole, 1988), firms in a vertically differentiated industry do not occupy identical positions to avoid marginal cost pricing. The above results show that, in the quadratic duopoly case, the strategic effect[139] of reducing / increasing quality in order to soften the competitor's pricing behavior is so strong that the equilibrium qualities lie even outside the range of preferred qualities.

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137 β is assumed to be sufficiently large so that the low quality is greater or equal to zero. See the Appendix.

138 In Tirole's (1988) model, all consumers would choose a product of infinite quality when sold at unit cost.

139 in the terminology of Tirole (1988); the effect of a change in q_2 on firm 2's profit can be decomposed as follows: $\dfrac{d\pi_2(q_1, q_2, p_1^*(q_1,q_2), p_2^*, (q_1,q_2))}{dq_2} = \dfrac{\partial \pi_2}{\partial q_2} + \dfrac{\partial \pi_2}{\partial p_1}\dfrac{\partial p_1^*(q_1,q_2)}{\partial q_2}$; the first term is called the direct effect, the latter one is the strategic effect

Now consider the more general non-linear case, where δ may be different from 2. The main result of the analysis in this chapter is summarized by the following Proposition:

Proposition 1: In an interior covered market equilibrium, for $\delta > 2$, the profit of the high-quality provider is larger than the one of the low firm, while for $\delta < 2$, the low firm earns the larger profit.

Proof: See the Appendix.

The idea that under non-linear preferences and costs, the high-quality advantage can be reversed may not be very surprising: suppose the quality levels of the two firms were exogenously fixed; if the unit cost of production increases fast enough in quality (that is, faster than the consumers' willingness to pay), why should we expect the high-quality provider to earn the higher profit? What is surprising about the above result, however, is that for a high degree of non-linearity, the high-quality advantage is actually restored!

To understand the intuition behind this result, we have to take into consideration the strategic effects of a change in the degree of non-linearity. If δ is increased (say, starting from $\delta = 2$), the difference between the unit cost and the consumers' willingness to pay is reduced at higher quality levels. The negative effect upon firm 2's profit can partially be cushioned by lowering q_2. The negative side-effect of this is that firm 1's pricing behavior becomes more aggressive. Since firm 1 anticipates firm 2's choice of a lower quality level, firm 1 has an additional incentive to choose a lower quality level itself. This, however, positively affects firm 2's profit, while firm 1 suffers from the reduced level of q_2. This leads to the high-quality advantage. In sum: there are two effects of an increased degree of non-linearity of $f(.)$: Firstly, a reduced margin between the consumers' willingness-to-pay and unit cost as a direct effect, and secondly, a commitment effect: when the exponent in $f(.)$ is sufficiently high, firm 2 is effectively committed not to choose very high quality levels, and firm 1, anticipating this, will choose an even smaller level of q_1.

Figure 1 shows that both firms suffer from an increased degree of non-linearity, but – as a result of the strategic interaction of the two firms – the low-quality provider suffers even more than the high firm. This is what drives the above result.

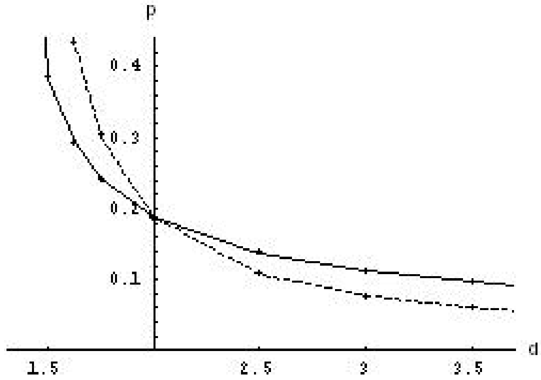

Figure 1: Equilibrium profits π_1 and π_2 for different exponents δ ($\alpha = \beta = 1$; π_1: dashed)

For lower values of δ, the situation is reversed, and a low-quality-advantage is obtained.[140] For $\delta \to 1$, Tirole's linear model is obtained. For this model, however, a high-quality-advantage is always obtained. To understand the difference between the model presented above, for δ near 1, and Tirole's linear model, note, that Tirole assumes an exogenous upper bound to the set of feasible quality choices (otherwise, the high-quality provider would choose $q_2 \to \infty$), and a lower bound to assure market coverage. Now it should be clear from the discussion above that an exogenous upper bound to quality serves as a commitment device for the high firm not to choose very high quality levels, and this can induce a high-quality-advantage, while a lower bound to quality can lead to a low-quality-advantage. The fact that Tirole (1988) predicts an unambiguous high-quality-advantage (despite the lower bound to quality) relies on the restrictive assumptions made by the author.[141]

To see that an exogenous bound to quality can induce an artificial high- / low-quality advantage, consider the following example. Let $\delta = 2$, $\alpha = 3/8$, and $\beta = 1$. One obtains in equilibrium: $q_1 = 1$, $q_2 = 3$, and $\pi_1 = \pi_2 = 1/2$. Now suppose there is a lower bound to

[140] For an intuitive explanation of the low-quality-advantage, consider the plot (over q) of the willingness-to-pay (wtp) minus unit cost for the average consumer ($t = 1/2$), for $\delta < 2$. It increases steeply at low levels of q and then declines slowly. q_1 is located in the increasing range, q_2 in the decreasing one. What matters for the strategic determination of qualities the *slope* (the rate of change in q of wtp minus unit cost), while the *level* at q_1 and q_2 matters for profit. Since the curve is steep at the low end, a high slope is reached already at high levels of the curve, while a comparable slope (in absolute value) in the decreasing range is reached at a lower level.

[141] in particular, that the unit cost is independent of quality, and that utility is linear in quality; under these assumptions, the difference between the willingness-to-pay and unit cost always increases in quality

quality[142] at $q = 1.5$. One obtains in equilibrium: $q_1 = 1.5$, $q_2 = 3.166$, $\pi_1 = 0.567$, $\pi_2 = 0.289$, so $\pi_1 / \pi_2 = 1.96$. The lower bound to quality induces a pronounced low-quality advantage. Firm 1's profit even increases in absolute terms (not just relative to π_2) when the lower bound is introduced. The firm benefits from the bound to quality as a commitment device.

5.4. CONCLUDING REMARKS

In linear models of vertically differentiated markets, an exogenous upper bound to quality must be imposed to avoid the problem of an infinitely high quality choice. Furthermore, a lower bound is assumed if the focus is on covered market equilibria. An exogenous bound to quality confers commitment power to the firms not to choose a very high / very low quality level. This can induce an (artificial) high- or low-quality advantage. The fact that Tirole (1988) always obtains a high-quality-advantage relies on the specific assumptions of the model, in particular the quality - independent unit cost and the linear utility - quality relation. It seems more natural to assume that utility is concave and / or that the unit cost of production is convex in quality. As a result, one can either obtain a high- or a low-quality advantage, depending on the degree of non-linearity assumed in the model. For a low degree of non-linearity, the model presented in this chapter predicts a low-quality-advantage, while the high-quality-advantage is restored for higher degrees of non-linearity (a quadratic relation marks the turning point between these two cases). A high degree of non-linearity is similar to an exogenous upper bound to quality, as it confers commitment power to the high firm not to choose very high quality levels. The existence of a high- or a low-quality advantage, thus, depends on the nature of strategic interaction, which is related to the degree of non-linearity.

5.5. APPENDIX

Proof of Proposition 1:

First, it will be shown that the parameterization of the model can be simplified without affecting the value of the profit ratio. It turns out that this ratio is independent of one of the remaining parameters. This simplifies the proof considerably.

For an ease of notation, "Δ" is used for differences between qualities or functional values of qualities. E.g., Δf stands for $f_2 - f_1 = f(q_2) - f(q_1)$.

[142] A lower bound to quality can be interpreted as a minimum quality standard. See Ronnen (1991) or Crampes and Hollander (1995).

From (8), one obtains: $\Delta f = \beta\Delta q - \alpha(q_2^{\delta} - q_1^{\delta})$. Using this and (8), the FOCs (6) become:

$$\alpha\delta q_1^{\delta-1} = \frac{1}{2}\beta - \frac{1}{2} + \alpha\frac{(q_2^{\delta} - q_1^{\delta})}{2\Delta q} \quad \text{and} \quad \alpha\delta q_2^{\delta-1} = \frac{1}{2}\beta + 1 + \alpha\frac{(q_2^{\delta} - q_1^{\delta})}{2\Delta q} \tag{11}$$

Using (3) and the expression for Δf, one obtains for (the square root of) the profit ratio:

$$\sqrt{\frac{\pi_1}{\pi_2}} = \frac{(1-\beta)\Delta q + \alpha(q_2^{\delta} - q_1^{\delta})}{(2+\beta)\Delta q - \alpha(q_2^{\delta} - q_1^{\delta})} \tag{12}$$

In the following, it is shown that the profit ratio is independent of α.

First, consider the expression $\alpha(q_2^{\delta} - q_1^{\delta})$ that appears in (12). This can be rewritten in a more convenient way. Multiply equations (11) by, respectively, q_1 and q_2 to obtain: $\alpha\delta q_1^{\delta} = \left(\frac{\beta}{2} - \frac{1}{2}\right)q_1 + \frac{\alpha(q_2^{\delta} - q_1^{\delta})}{2\Delta q}q_1$ and $\alpha\delta q_2^{\delta} = \left(\frac{\beta}{2} + 1\right)q_2 + \frac{\alpha(q_2^{\delta} - q_1^{\delta})}{2\Delta q}q_2$. Subtracting these two equations and rearranging yields:

$$\alpha(q_2^{\delta} - q_1^{\delta}) = \frac{1}{2\delta - 1}\left(\beta\Delta q + 2q_2 + q_1\right) \tag{13}$$

Using (13) in (12), both numerator and denominator of the profit ratio become linear in q_1 and q_2, and do not explicitly contain the parameter α. However, the equilibrium quality levels q_1 and q_2 are functions of the parameters: $q_1 = q_1(\alpha,\beta,\delta)$, $q_2 = q_2(\alpha,\beta,\delta)$.

Now define: $\tilde{q}_1(\alpha,\beta,\delta) \equiv \alpha^k q_1(\alpha,\beta,\delta)$, and $\tilde{q}_2(\alpha,\beta,\delta) \equiv \alpha^k q_2(\alpha,\beta,\delta)$, where k is a constant. Using these definitions, the FOCs (11) can be rewritten as: $\alpha^{k(1-\delta)+1}\delta\tilde{q}_1^{\delta-1} = \frac{1}{2}\beta - \frac{1}{2} + \alpha^{k(1-\delta)+1}\frac{(\tilde{q}_2^{\delta} - \tilde{q}_1^{\delta})}{2\Delta\tilde{q}}$ and $\alpha^{k(1-\delta)+1}\delta\tilde{q}_2^{\delta-1} = \frac{1}{2}\beta + 1 + \alpha^{k(1-\delta)+1}\frac{(\tilde{q}_2^{\delta} - \tilde{q}_1^{\delta})}{2\Delta\tilde{q}}$. The idea is to choose k such that α disappears in the FOCs. This is the case for $k \equiv \frac{1}{\delta - 1}$.

$\tilde{q}_1$ and $\tilde{q}_2$ are, then, only functions of β and δ. From the definition of $\tilde{q}_1$ and $\tilde{q}_2$, it, thus, follows that:

$$q_1(\alpha,\beta,\delta) = \alpha^{-\frac{1}{\delta-1}}\tilde{q}_1(\beta,\delta) \quad \text{and} \quad q_2(\alpha,\beta,\delta) = \alpha^{-\frac{1}{\delta-1}}\tilde{q}_2(\beta,\delta) \tag{14}$$

Using (13) and (14) in (12) shows that α cancels out. The profit ratio has, thus, been shown to be independent from α.

There is a minimum value for β, denoted by $\beta_{\min}$, that assures that the FOCs have at least one solution for which $q_1 \geq 0$ holds. Set $q_1 \equiv 0$. (11) simplifies to: $\alpha\left(\delta - \frac{1}{2}\right)q_2^{\delta-1} = 1 + \frac{\beta}{2}$ and $q_2^{\delta-1} = \frac{1-\beta}{\alpha}$. Combining these two equations yields (note, that α cancels out):

$$\beta_{\min}(\delta) = 1 - \frac{3}{2\delta} \tag{15}$$

For example: $\delta = 1.5$ yields $\beta_{\min} = 0$, $\delta = 2$ yields $\beta_{\min} = \frac{1}{4}$, and $\beta_{\min} = \frac{1}{2}$ for $\delta = 3$.

It remains to be shown that, for $\delta < 2$ ($\delta > 2$), $\frac{\pi_1}{\pi_2} > 1$ ($\frac{\pi_1}{\pi_2} < 1$) holds for all β. (5) is a condition for $\frac{\pi_1}{\pi_2} > 1$. Using (8) and (13), this condition can be written as:

$$r > z\text{, with } r \equiv q_2/q_1 \text{ and } z \equiv \frac{2(\delta-1)\beta + \delta + \frac{1}{2}}{2(\delta-1)\beta + \delta - \frac{5}{2}} \tag{16}$$

That is, the condition for $\frac{\pi_1}{\pi_2} > 1$ has been re-written as a condition on the ratio of equilibrium quality levels.

The FOCs (11) can be combined into a single non-linear equation that determines the equilibrium value for r. Dividing the equations by each other, and using (13), one obtains:

$$r^{\delta-1} = \frac{2\delta(\beta+2)(r-1)+3}{2\delta(\beta-1)(r-1)+3r} \tag{17}$$

It must be shown that $r > z$ holds for $1 < \delta < 2$, $r = z$ for $\delta = 2$, and $r < z$ for $\delta > 2$. The second claim ($r = z$ for $\delta = 2$) is easily confirmed. For $\delta = 2$, r simply equals the critical value for r: z for all β. To show that the other two claims hold, one may insert z into the equation that determines r. It, then, suffices to show that the resulting equation in β and δ has no solution for $\delta \neq 2$, because by continuity, then, r is larger (smaller) than z for all β and all $\delta \in (1,2)$ ($\delta \in (2,\infty)$). Replacing r by z, one obtains:

$$\left(\frac{2(\delta-1)\beta + \delta + \frac{1}{2}}{2(\delta-1)\beta + \delta - \frac{5}{2}}\right)^{\delta-1} = \frac{\beta + \frac{5}{4}}{\beta - \frac{1}{4}} \tag{18}$$

For a given δ, the left- and the right-hand-side (LHS, RHS) of this equation can be seen as two curves that intersect at a solution of the equation. For $\delta = 2$, both curves are identical. Note, that the RHS of (18) is independent of δ. To show that for $\delta \neq 2$, the equation has no solution, it, thus, suffices to show that the set of curves defined by the LHS do not intersect each other. That is, starting from $\delta = 2$ where both curves are identical, if δ is increased (or decreased), the curve of the LHS is shifted away from the one of the RHS for all β. Here, this is shown indirectly by analyzing the *slope* of the LHS at a fixed β for different values of δ. Define the function $h(\beta,\delta)$ as the natural logarithm of the LHS:

$$h(\beta,\delta) = (\delta-1)\ln\left(\frac{\beta+\delta_2}{\beta+\delta_1}\right), \text{ where } \delta_2 \equiv \frac{\delta + \frac{1}{2}}{2(\delta-1)} \text{ and } \delta_1 \equiv \frac{\delta - \frac{5}{2}}{2(\delta-1)} \tag{19}$$

Since the logarithm is a strict monotone increasing transformation, it suffices to analyze the slope of h. Taking the derivative w.r.t. β, one obtains: $h'(\delta,\beta) = -\frac{(\delta-1)(\delta_2-\delta_1)}{(\beta+\delta_2)(\beta+\delta_1)}$. It must be shown that the absolute value of this slope decreases as δ increases. That is, for any $\tilde{\delta} > \delta > 1$, it must hold that: $\left|h'(\beta,\delta)\right| > \left|h'(\beta,\tilde{\delta})\right|$. Using the condition that $\beta+\delta_1 > 0$ (otherwise, the denominator in LHS, and, thus, r is negative, which implies $q_1 < 0$), after some re-arrangements, the condition $\left|h'(\beta,\delta)\right| > \left|h'(\beta,\tilde{\delta})\right|$ simplifies to $\tilde{\delta} > \delta$, which is true by assumption. Therefore, the slope of the curve defined by the LHS is decreasing in δ. Noting that $\lim_{\beta\to\infty} r(\beta) = \lim_{\beta\to\infty} z(\beta) = 1$ (which is easy to verify), the equation to be analyzed has, thus, been shown to have no solution for $\delta \neq 2$. (Rigorously, one must also show that LHS does not intersect with the lower branch of RHS, since RHS has a polar point. It can be shown that the limiting curve for $\delta \to \infty$ of LHS is given by: $e^{3/(2\beta+1)}$, which lies strictly above the lower branch of RHS.)

Outlook

The reader may wonder, why in this work, there are chapters on markets with demand rigidity, and chapters on vertical differentiation with apparently few conceptual intersection points. The choice of these topics was motivated by a broader vision of markets with demand rigidity, where firms choose quality endogenously and invest in R&D to develop higher-quality products. This vision combines ideas from all chapters of this work. The main idea is briefly sketched in the following.

Suppose, a firm moves up on the quality ladder[143], and reaches the quality level of its competitor that dominated the market in the past. In existing models with endogenous quality growth, authors generally neglect the possibility of demand rigidity, e.g. caused by incomplete consumer information. Therefore, market shares immediately adjust to the new conditions after the innovation, and the previous follower reaches an equal market share as the competitor. However, under demand rigidity, market shares are sticky and do not immediately adjust. Therefore, the previous leader may still serve a large fraction of market demand after the innovation. This reduces the follower's incentives to innovate, and puts the leader in a fairly "secure" position.

One of the questions that one could ask in this context is whether there exist dynamically stable constellations with leaders and followers with different market shares. E.g., there may be a large technological leader chased by several smaller followers, or a high-priced leader at the technological frontier and a large follower that serves most of the market. Pioneering work in this field has been done by Fishman and Rob (2003). The authors show that, in a market where consumers are locked-in at their previous supplier due to search costs, firms with a large customer base earn higher profits and invest more in R&D than smaller firms. However, the authors assume that firms only compete for new consumers who arrive in the market, while older consumers are fully locked-in. In a similar model, Rob and Fishman (2005) introduce word-of-mouth. Information about the past performance of firms, thus, spreads only gradually in the economy. However, also in this work, the authors abstract largely from price competition by assuming that consumers are locked-in at their previous supplier. This could be a good starting point for future research.

Apart from this, there is a weakness of the models introduced in this work that shall briefly be discussed here. But it is important to note that this is not a weakness that is specific to these models. It equally concerns a large number of other models, e.g. from the search literature.

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[143] See Grossman and Helpman (1991).

Confronting this weakness, and trying to resolve it, could be another interesting starting point for future research.
This weakness is, that these models neglect the importance that retailers have in the determination of market outcomes. E.g., if consumers are not fully informed about the existence or the offers of the different firms in a market – as was assumed in chapter 2 and 3 of this work – then shops play an important role because they make a pre-selection of the products with respect to quality and price. This may, indeed, be one of the main functions of shops in general, possibly eliminating much of the quality uncertainty that might otherwise prevail in a market economy.[144] A "complete" model of a market should, thus, account for the role that retailers play as information intermediaries and price setters. The explicit inclusion of firms *and* retailers in oligopoly models with incomplete consumer information is a real challenge for future research.
Authors from the search literature often refer to *shops* rather than firms at the supply side of their models. In this work, I have chosen not to adopt this notion. The reason for this is that shops usually offer a variety of products. Therefore, they do not set a single price, as was assumed for the "firms" in chapter 2 and 3. They set a large number of prices, and usually offer a low price from time to time for certain products but not for others. Therefore, the average price of a basket of goods may be roughly constant over time.
Another complication arises when firms do not offer a single product, but a set of different product versions. This complicates the picture, because the demands that a firm faces for the different product versions are not independent. Multi-product firms that are active in several markets may also find it easier to establish a reputation for offering goods of a certain quality level than single-product firms, because consumers purchase from them more frequently.

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[144] Stiglitz (1989) writes: "The existence of imperfect information gives rise to a demand for information… (this) may give rise to firms which specialize in the production and dissemination of information… Indeed, on can view wholesalers, or even retailers as being largely information intermediaries."

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